

Oakfield Lodge School Mathematics Year 9 Scheme of Learning 2021-2022

Unit	Lessons	Key 'Build a Mathematician' (BAM) Indicators	Essential knowledge
Numbers and the number system	12	<ul style="list-style-type: none"> Use positive integer powers and associated real roots Apply the four operations with decimal numbers Write a quantity as a fraction or percentage of another Use multiplicative reasoning to interpret percentage change Add, subtract, multiply and divide with fractions and mixed numbers Check calculations using approximation, estimation or inverse operations Simplify and manipulate expressions by collecting like terms Simplify and manipulate expressions by multiplying a single term over a bracket Substitute numbers into formulae Solve linear equations in one unknown Understand and use lines parallel to the axes, $y = x$ and $y = -x$ Calculate surface area of cubes and cuboids Understand and use geometric notation for labelling angles, lengths, equal lengths and parallel lines 	<ul style="list-style-type: none"> Know the first 6 cube numbers Know the first 12 triangular numbers Know the symbols =, ≠, <, >, ≤, ≥ Know the order of operations including brackets Know basic algebraic notation Know that area of a rectangle = $l \times w$ Know that area of a triangle = $b \times h \div 2$ Know that area of a parallelogram = $b \times h$ Know that area of a trapezium = $((a + b) \div 2) \times h$ Know that volume of a cuboid = $l \times w \times h$ Know the meaning of faces, edges and vertices Know the names of special triangles and quadrilaterals Know how to work out measures of central tendency Know how to calculate the range
Calculating	16		
Checking, approximating and estimating	3		
Counting and comparing	9		
Visualising and constructing	4		
Investigating properties of shapes	5		
Algebraic proficiency: tinkering	8		
Exploring fractions, decimals and percentages	4		
Proportional reasoning	4		
Pattern sniffing	3		
Measuring space	7		
Investigating angles	3		
Calculating fractions, decimals and percentages	15		
Solving equations and inequalities	5		
Calculating space	5		
Mathematical movement	7		
Presentation of data	6		
Measuring data	7		
Total:	123	Stage 7 BAM Progress Tracker Sheet	

Maths Calendar

Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12	Week 13
Numbers and the number system 7M1 BAM			Calculating 7M2 BAM					Checking 7M6 BAM	Counting and comparing			Visualising etc 7M13 BAM
Week 14	Week 15	Week 16	Week 17	Week 18	Week 19	Week 20	Week 21	Week 22	Week 23	Week 24	Week 25	Week 26
Assess and enrich	Properties of shapes	Algebraic proficiency: tinkering 7M7 BAM, 7M8 BAM, 7M9 BAM			Exploring FDP 7M3 BAM	Prop'l reasoning	Patterns	Measuring space		Angles	Calculating FDP	
Week 27	Week 28	Week 29	Week 30	Week 31	Week 32	Week 33	Week 34	Week 35	Week 36	Week 37	Week 38	Week 39
Assessment	Calculating FDP cont'd 7M4 BAM, 7M5 BAM		Solving equations 7M10 BAM	Calculating space 7M12 BAM		Mathematical movement 7M11 BAM		Presentation of data		Measuring data		Assessment

Key concepts (GCSE subject content statements)

The Big Picture: [Number and Place Value progression map](#)

- use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor and lowest common multiple
- use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions

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Possible themes		Possible key learning points	
<ul style="list-style-type: none"> • Solve problems using common factors and highest common factors • Exploring prime numbers • Solve problems using common multiples and lowest common multiples • Explore powers and roots 		<ul style="list-style-type: none"> • Find prime numbers and test numbers to see if they are prime • Find common factors of numbers • Find the highest common factor of numbers in simple cases, including co-prime examples • Find common multiples of numbers • Recognise and solve problems involving the lowest common multiple • Use linear (arithmetic) number patterns to solve problems • Recognise and use triangular numbers • Recognise and use square and cube numbers • Read, write and evaluate powers • Define and find square roots (including using the $\sqrt{\quad}$ symbol) • Define and find cube roots (including using the $\sqrt[3]{\quad}$ symbol), including the use of a scientific calculator • Define and find other roots (including using the $\sqrt{\quad}$ symbol), including the use of a scientific calculator 	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> • Know how to find common multiples of two given numbers • Know how to find common factors of two given numbers • Recall multiplication facts to 12×12 and associated division facts <p>Bring on the Maths*: Moving on up! Number and Place Value: #6</p>	((Lowest) common) multiple and LCM ((Highest) common) factor and HCF Power (Square and cube) root Triangular number, Square number, Cube number, Prime number Linear sequence, Arithmetic sequence Notation Index notation: e.g. 5^3 is read as '5 to the power of 3' and means '3 lots of 5 multiplied together' Radical notation: e.g. $\sqrt{49}$ is generally read as 'the square root of 49' and means 'the positive square root of 49'; $\sqrt[3]{8}$ means 'the cube root of 8'	Pupils need to know how to use a scientific calculator to work out powers and roots. Note that while the square root symbol ($\sqrt{\quad}$) refers to the positive square root of a number, every positive number has a negative square root too. NCETM: Departmental workshop: Index Numbers NCETM: Glossary Common approaches <i>The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.</i> <i>Every classroom has a set of number classification posters on the wall</i>	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> • When using Eratosthenes sieve to identify prime numbers, why is there no need to go further than the multiples of 7? If this method was extended to test prime numbers up to 200, how far would you need to go? Convince me. • Kenny says '20 is a square number because $10^2 = 20$'. Explain why Kenny is wrong. Kenny is partially correct. How could he change his statement so that it is fully correct? • Always / Sometimes / Never: The lowest common multiple of two numbers is found by multiplying the two numbers together. 	KM: Perfect numbers : includes use of factors, primes and powers KM: Exploring primes activities : Factors of square numbers; Mersenne primes; LCM sequence; n^2 and $(n + 1)^2$; n^2 and $n^2 + n$; $n^2 + 1$; $n! + 1$; $n! - 1$; $x^2 + x + 41$ KM: Use the method of Eratosthenes' sieve to identify prime numbers, but on a grid 6 across by 17 down instead. What do you notice? KM: Square number puzzle KM: History and Culture: Goldbach's Conjectures NRICH: Factors and multiples NRICH: Powers and roots Learning review KM: 7M1 BAM Task	<ul style="list-style-type: none"> • Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1' • A common misconception is to believe that $5^3 = 5 \times 3 = 15$ • See pedagogical note about the square root symbol too 	

Key concepts (GCSE subject content statements)

The Big Picture: [Calculation progression map](#)

- understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
- apply the four operations, including formal written methods, to integers and decimals
- use conventional notation for priority of operations, including brackets
- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)

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Possible themes	Possible key learning points	
<ul style="list-style-type: none"> • Exploring place value • Exploring written methods of calculation • Calculating with decimals • Know and apply the correct order of operations 	<ul style="list-style-type: none"> • Multiply a positive integer by a power of 10 • Multiply a decimal by a power of 10 • Divide a positive integer by a power of 10 • Divide a decimal by a power of 10 • Add numbers up to six-digits using a formal written method • Add decimals with the same, and different, number of decimal places • Subtract numbers up to six-digits using a formal written method • Subtract decimals with the same, and different, number of decimal places 	<ul style="list-style-type: none"> • Multiply a number up to four-digits by a one or two-digit number using a formal written method • Transform a multiplication involving decimals to a corresponding multiplication with integers • Multiply a large integer up to four-digits by a decimal of up to 2dp using integer multiplication • Divide a number up to four-digits by a one or two-digit number using a formal written method • Use a formal method to divide a decimal by an integer < 10 • Use a formal method to divide a decimal by an integer greater than 10 • Transform a calculation involving the division of decimals to an equivalent division involving integers • Apply the order of operations to multi-step calculations involving up to four operations and brackets
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Fluently recall multiplication facts up to 12×12 • Fluently apply multiplication facts when carrying out division • Know the formal written method of long multiplication • Know the formal written method of short division • Know the formal written method of long division • Convert between an improper fraction and a mixed number <p>Bring on the Maths*: Moving on up! Calculating: #2, #3, #4, #5 Fractions, decimals & percentages: #6, #7 Solving problems: #2</p>	Improper fraction Top-heavy fraction Mixed number Operation Inverse Long multiplication Short division Long division Remainder	<p>Note that if not understood fully, BIDMAS can give the wrong answer to a calculation; e.g. $6 - 2 + 3$. The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.</p> <p>KM: Progression: Addition and Subtraction, Progression: Multiplication and Division and Calculation overview NCETM: Departmental workshop: Place Value NCETM: Subtraction, Multiplication, Division, Glossary</p> <p>Common approaches <i>All classrooms display a times table poster with a twist</i> <i>Long multiplication is promoted as the 'most efficient method'.</i> <i>Short division is promoted as the 'most efficient method'.</i> <i>If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.</i></p>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Jenny says that $2 + 3 \times 5 = 25$. Kenny says that $2 + 3 \times 5 = 17$. Who is correct? How do you know? • Find missing digits in otherwise completed long multiplication / short division calculations • Show me a calculation that is connected to $14 \times 26 = 364$. And another. And another ... 	KM: Long multiplication template KM: Dividing (lots) KM: Interactive long division KM: Misplaced points KM: 4 to 1 challenge KM: Maths to Infinity: Multiplying and dividing NRICH: Cinema Problem NRICH: Funny factorisation NRICH: Skeleton NRICH: Long multiplication	<ul style="list-style-type: none"> • The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations. • Pupils may incorrectly apply place value when dividing by a decimal for example by making the answer 10 times bigger when it should be 10 times smaller. • Some pupils may have inefficient methods for multiplying and dividing numbers.
	<p>Learning review KM: 7M2 BAM Task</p>	

Key concepts (GCSE subject content statements)

The Big Picture: [Number and Place Value progression map](#)

- round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures)
- estimate answers; check calculations using approximation and estimation, including answers obtained using technology
- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)

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Possible themes		Possible key learning points
<ul style="list-style-type: none"> • Explore ways of approximating numbers • Explore ways of checking answers 		<ul style="list-style-type: none"> • Round a number to a specified number of decimal places • Round a number to one significant figure • Estimate calculations by rounding numbers to one significant figure
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000, 100 000 or 1 000 000 • Approximate any number with one or two decimal places by rounding to the nearest whole number • Approximate any number with two decimal places by rounding to the one decimal place • Simplify a fraction by cancelling common factors 	<p>Approximate (noun and verb) Round Decimal place Check Solution Answer Estimate (noun and verb) Order of magnitude Accurate, Accuracy Significant figure Cancel Inverse Operation</p> <p>Notation The approximately equal symbol (\approx) Significant figure is abbreviated to 's.f.' or 'sig fig'</p>	<p>Pupils should be able to estimate calculations involving integers and decimals. Also see big pictures: Calculation progression map and Fractions, decimals and percentages progression map NCETM: Glossary</p> <p>Common approaches <i>All pupils are taught to visualise rounding through the use a number line</i></p>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Convince me that 39 652 rounds to 40 000 to one significant figure • Convince me that 0.6427 does <u>not</u> round to 1 to one significant figure • What is wrong: $\frac{11 \times 28.2}{0.54} \approx \frac{10 \times 30}{0.5} = 150$. How can you correct it? 	<p>KM: Approximating calculations KM: Stick on the Maths: CALC6: Checking solutions</p> <p>Learning review KM: 7M6 BAM Task</p>	<ul style="list-style-type: none"> • Some pupils may truncate instead of round • Some pupils may round down at the half way point, rather than round up. • Some pupils may think that a number between 0 and 1 rounds to 0 or 1 to one significant figure • Some pupils may divide by 2 when the denominator of an estimated calculation is 0.5

Key concepts (GCSE subject content statements)

The Big Picture: [Number and Place Value progression map](#)

- order positive and negative integers, decimals and fractions
- use the symbols =, ≠, <, >, ≤, ≥

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Possible themes	Possible key learning points	
<ul style="list-style-type: none"> • Comparing numbers • Ordering integers and decimals • Ordering fractions • Ordering integers, decimals and fractions (including mixed numbers) • Using comparison symbols in algebraic contexts 	<ul style="list-style-type: none"> • Use the signs <, > and = to compare numbers • Use a compound inequality to compare three or more numbers (e.g. $-1 < 0.5 < 4$) • Order a set of integers • Order a set of decimals • Order a set of integers and decimals • Order fractions with the same denominator or denominators are a multiple of each other • Order fractions where the denominators are not multiples of each other • Order mixed numbers and fractions • Order a combination of integers, decimals, fractions and mixed numbers 	
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Understand that negative numbers are numbers less than zero • Order a set of decimals with a mixed number of decimal places (up to a maximum of three) • Order fractions where the denominators are multiples of each other • Order fractions where the numerator is greater than 1 • Know how to simplify a fraction by cancelling common factors 	Positive number Negative number Integer Numerator Denominator Notation The 'equals' sign: = The 'not equal' sign: ≠ The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to)	Zero is neither positive nor negative. The set of integers includes the natural numbers {1, 2, 3, ...}, zero (0) and the 'opposite' of the natural numbers {-1, -2, -3, ...}. Pupil must use language correctly to avoid reinforcing misconceptions: for example, 0.45 should never be read as 'zero point forty-five'; $5 > 3$ should be read as 'five is greater than 3', not '5 is bigger than 3'. Ensure that pupils read information carefully and check whether the required order is smallest first or greatest first. The equals sign was designed by Robert Recorde in 1557 who also introduced the plus (+) and minus (-) symbols. NCETM: Glossary Common approaches <i>Teachers use the language 'negative number' to avoid future confusion with calculation that can result by using 'minus number'</i> <i>Every classroom has a negative number washing line on the wall</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Jenny writes down $0.400 > 0.58$. Kenny writes down $0.400 < 0.58$. Who do you agree with? Explain your answer. • Find a fraction which is greater than $\frac{3}{5}$ and less than $\frac{7}{8}$. And another. And another ... • Convince me that $-15 < -3$ 	KM: Inequality KM: Farey Sequences KM: Decimal ordering cards 2 KM: Maths to Infinity: Fractions, decimals and percentages KM: Maths to Infinity: Directed numbers NRICH: Greater than or less than? YouTube: The Story of Zero	<ul style="list-style-type: none"> • Some pupils may believe that 0.400 is greater than 0.58 • Pupils may believe, incorrectly, that: <ul style="list-style-type: none"> - A fraction with a larger denominator is a larger fraction - A fraction with a larger numerator is a larger fraction - A fraction involving larger numbers is a larger fraction • Some pupils may believe that -6 is greater than -3. For this reason ensure pupils avoid saying 'bigger than'

Key concepts (GCSE subject content statements)

The Big Picture: [Properties of Shape progression map](#)

- use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries
- use the standard conventions for labelling and referring to the sides and angles of triangles
- draw diagrams from written description

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Possible themes	Possible key learning points
<ul style="list-style-type: none"> • Interpret geometrical conventions and notation • Apply geometrical conventions and notation <p>Bring on the Maths+: Moving on up! Properties of shapes: #3, #4</p>	<ul style="list-style-type: none"> • Identify line and rotational symmetry in polygons • Understand and use labelling notation for lengths and angles • Use ruler and protractor to construct triangles, and other shapes, from written descriptions • Use ruler and compasses to construct triangles when all three sides known

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Use a ruler to measure and draw lengths to the nearest millimetre • Use a protractor to measure and draw angles to the nearest degree 	Edge, Face, Vertex (Vertices) Plane Parallel Perpendicular Regular polygon Rotational symmetry Notation The line between two points A and B is AB The angle made by points A, B and C is $\angle ABC$ The angle at the point A is \hat{A} Arrow notation for sets of parallel lines Dash notation for sides of equal length	NCETM: Departmental workshop: Constructions The equals sign was designed (by Robert Recorde in 1557) based on two equal length lines that are equidistant NCETM: Glossary Common approaches <i>Dynamic geometry software to be used by all students to construct and explore dynamic diagrams of perpendicular and parallel lines.</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Given SSS, how many different triangles can be constructed? Why? Repeat for ASA, SAS, SSA, AAS, AAA. • Always / Sometimes / Never: to draw a triangle you need to know the size of three angles; to draw a triangle you need to know the size of three sides. • Convince me that a hexagon can have rotational symmetry with order 2. 	KM: Shape work (selected activities) KM: Rotational symmetry NRICH: Notes on a triangle Learning review KM: 7M13 BAM Task	<ul style="list-style-type: none"> • Two line segments that do not touch are perpendicular if they would meet at right angles when extended • Pupils may believe, incorrectly, that: <ul style="list-style-type: none"> - perpendicular lines have to be horizontal / vertical - only straight lines can be parallel - all triangles have rotational symmetry of order 3 - all polygons are regular

Key concepts (GCSE subject content statements)

The Big Picture: [Properties of Shape progression map](#)

- identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
- derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

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Possible themes		Possible key learning points
<ul style="list-style-type: none"> • Investigate the properties of 3D shapes • Explore quadrilaterals • Explore triangles 		<ul style="list-style-type: none"> • Know the connection between faces, edges and vertices in 3D shapes • Recognise and use nets of 3D shapes • Know and solve problems using the properties and definitions of triangles • Know and solve problems using the properties and definitions of special types of quadrilaterals (including diagonals) • Know and solve problems using the properties of other plane figures
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Know the names of common 3D shapes • Know the meaning of face, edge, vertex • Understand the principle of a net • Know the names of special triangles • Know the names of special quadrilaterals • Know the meaning of parallel, perpendicular • Know the notation for equal sides, parallel sides, right angles <p>Bring on the Maths+: Moving on up! Properties of shapes: #1, #2</p>	Face, Edge, Vertex (Vertices) Cube, Cuboid, Prism, Cylinder, Pyramid, Cone, Sphere Quadrilateral Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus Delta, Arrowhead Diagonal Perpendicular Parallel Triangle Scalene, Right-angled, Isosceles, Equilateral Notation Dash notation to represent equal lengths in shapes and geometric diagrams Right angle notation	Ensure that pupils do not use the word 'diamond' to describe a kite, or a square that is 45° to the horizontal. 'Diamond' is not the mathematical name of any shape. A cube is a special case of a cuboid and a rhombus is a special case of a parallelogram A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid. NCETM: Departmental workshop: 2D shapes NCETM: Glossary Common approaches <i>Every classroom has a set of triangle posters and quadrilateral posters on the wall</i> <i>Models of 3D shapes to be used by all students during this unit of work</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Show me an example of a trapezium. And another. And another ... • Always / Sometimes / Never: The number of vertices in a 3D shape is greater than the number of edges • Which quadrilaterals are special examples of other quadrilaterals? Why? Can you create a 'quadrilateral family tree'? • What is the same and what is different: Rhombus / Parallelogram? 	KM: Euler's formula KM: Visualising 3D shapes KM: Complete the net KM: Dotty activities : Shapes on dotted paper KM: What's special about quadrilaterals? Constructing quadrilaterals from diagonals and summarising results. NRICH: A chain of polyhedra NRICH: Property chart NRICH: Quadrilaterals game	<ul style="list-style-type: none"> • Some pupils may think that all trapezia are isosceles • Some pupils may think that a diagonal cannot be horizontal or vertical • Two line segments that do not touch are perpendicular if they would meet at right angles when extended. Therefore the diagonals of an arrowhead (delta) are perpendicular despite what some pupils may think • Some pupils may think that a square is only square if 'horizontal', and even that a 'non-horizontal' square is called a diamond • The equal angles of an isosceles triangle are not always the 'base angles' as some pupils may think

Key concepts (GCSE subject content statements)

The Big Picture: [Algebra progression map](#)

- understand and use the concepts and vocabulary of expressions, equations, formulae and terms
- use and interpret algebraic notation, including: ab in place of $a \times b$, $3y$ in place of $y + y + y$ and $3 \times y$, a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a/b in place of $a \div b$, brackets
- simplify and manipulate algebraic expressions by collecting like terms and multiplying a single term over a bracket
- where appropriate, interpret simple expressions as functions with inputs and outputs
- substitute numerical values into formulae and expressions
- use conventional notation for priority of operations, including brackets

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Possible themes

- Understand the vocabulary and notation of algebra
- Manipulate algebraic expressions
- Explore functions
- Evaluate algebraic statements

Possible key learning points

- Know the meaning of expression, term, formula, equation, function
- Know and use basic algebraic notation (the 'rules' of algebra)
- Simplify a simple expression by collecting like terms
- Simplify more complex expressions by collecting like terms
- Manipulate expressions by multiplying an integer over a bracket (the distributive law)
- Manipulate expressions by multiplying a single term over a bracket (the distributive law)
- Substitute positive numbers into expressions and formulae
- Given a function, establish outputs from given inputs and inputs from given outputs

Prerequisites

- Use symbols (including letters) to represent missing numbers
- Substitute numbers into worded formulae
- Substitute numbers into simple algebraic formulae
- Know the order of operations

Bring on the Maths+: Moving on up!

Algebra: #1

Mathematical language

Algebra
Expression, Term, Formula (formulae), Equation, Function, Variable
Mapping diagram, Input, Output
Represent
Substitute
Evaluate
Like terms
Simplify / Collect

Notation

See Key concepts (GCSE subject content statements) above

Pedagogical notes

Pupils will have experienced some algebraic ideas previously. Ensure that there is clarity about the distinction between representing a variable and representing an unknown.

Note that each of the statements $4x$, 42 and $4\frac{1}{2}$ involves a different operation after the 4, and this can cause problems for some pupils when working with algebra.

NCETM: [Algebra](#)
NCETM: [Glossary](#)

Common approaches

All pupils are expected to learn about the connection between mapping diagrams and formulae (to represent functions) in preparation for future representations of functions graphically.

Reasoning opportunities and probing questions

- Show me an example of an expression / formula / equation
- Always / Sometimes / Never: $4(g+2) = 4g+8$, $3(d+1) = 3d+1$, $a^2 = 2a$, $ab = ba$
- [What is wrong?](#)
- Jenny writes $2a + 3b + 5a - b = 7a + 3$. Kenny writes $2a + 3b + 5a - b = 9ab$. What would you write? Why?

Suggested activities

KM: [Pairs in squares](#). Prove the results algebraically.
KM: [Algebra rules](#)
KM: Use [number patterns](#) to develop the multiplying out of brackets
KM: [Algebra ordering cards](#)
KM: [Spiders and snakes](#). See the 'clouding the picture' approach
KM: [Maths to Infinity: Brackets](#)
NRICH: [Your number is ...](#)
NRICH: [Crossed ends](#)
NRICH: [Number pyramids](#) and [More number pyramids](#)

Learning review

KM: [7M7 BAM Task](#), [7M8 BAM Task](#), [7M9 BAM Task](#)

Possible misconceptions

- Some pupils may think that it is always true that $a=1$, $b=2$, $c=3$, etc.
- A common misconception is to believe that $a^2 = a \times 2 = a2$ or $2a$ (which it can do on rare occasions but is not the case in general)
- When working with an expression such as $5a$, some pupils may think that if $a=2$, then $5a = 52$.
- Some pupils may think that $3(g+4) = 3g+4$
- The convention of not writing a coefficient of 1 (i.e. '1x' is written as 'x' may cause some confusion. In particular some pupils may think that $5h - h = 5$

Key concepts (GCSE subject content statements)The Big Picture: [Fractions, decimals and percentages progression map](#)

- express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
- define percentage as 'number of parts per hundred'
- express one quantity as a percentage of another

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Possible themes		Possible key learning points
<ul style="list-style-type: none"> • Understand and use top-heavy fractions • Understand the meaning of 'percentage' • Explore links between fractions and percentages 		<ul style="list-style-type: none"> • Write one quantity as a fraction of another where the fraction is less than 1 • Write one quantity as a fraction of another where the fraction is greater than 1 • Write a percentage as a fraction • Write a quantity as a percentage of another
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Understand the concept of a fraction as a proportion • Understand the concept of equivalent fractions • Understand the concept of equivalence between fractions and percentages <p>Bring on the Maths+: Moving on up! Fractions, decimals & percentages: #1, #2</p>	Fraction Improper fraction Proper fraction Vulgar fraction Top-heavy fraction Percentage Proportion Notation Diagonal fraction bar / horizontal fraction bar	Describe $\frac{1}{3}$ as 'there are three equal parts and I take one', and $\frac{3}{4}$ as 'there are four equal parts and I take three'. Be alert to pupils reinforcing misconceptions through language such as 'the bigger half'. To explore the equivalency of fractions make several copies of a diagram with three-quarters shaded. Show that splitting these diagrams with varying numbers of lines does not alter the fraction of the shape that is shaded. NRICH: Teaching fractions with understanding NCETM: Teaching fractions NCETM: Departmental workshop: Fractions NCETM: Glossary Common approaches <i>All pupils are made aware that 'per cent' is derived from Latin and means 'out of one hundred'</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Jenny says '1/10 is the same as proportion as 10% so 1/5 is the same proportion as 5%.' What do you think? Why? • What is the same and what is different: 1/10 and 10% ... 1/5 and 20%? • Show this fraction as part of a square / rectangle / number line / ... 	KM: Crazy cancelling, silly simplifying NRICH: Rod fractions Learning review KM: 7M3 BAM Task	<ul style="list-style-type: none"> • A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape. • Pupils may not make the connection that a percentage is a different way of describing a proportion • Pupils may think that it is not possible to have a percentage greater than 100%

Key concepts (GCSE subject content statements)

The Big Picture: [Ratio and Proportion progression map](#)

- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part:part or part:whole ratio

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Possible themes		Possible key learning points	
<ul style="list-style-type: none"> • Understand and use ratio notation • Solve problems that involve dividing in a ratio 		<ul style="list-style-type: none"> • Describe a comparison of measurements or objects using ratio notation a:b • Simplify a ratio by cancelling common factors • Divide a quantity in two parts in a given part:part ratio • Divide a quantity in two parts in a given part:whole ratio 	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> • Find common factors of pairs of numbers • Convert between standard metric units of measurement • Convert between units of time • Recall multiplication facts for multiplication tables up to 12×12 • Recall division facts for multiplication tables up to 12×12 • Solve comparison problems <p>Bring on the Maths*: Moving on up! Ratio and proportion: #1</p>	Ratio Proportion Compare, comparison Part Simplify Common factor Cancel Lowest terms Unit Notation Ratio notation a:b for part:part or part:whole	Note that ratio notation is first introduced in this stage. When solving division in a ratio problems, ensure that pupils express their solution as two quantities rather than as a ratio. NCETM: The Bar Model NCETM: Multiplicative reasoning NCETM: Glossary Common approaches <i>All pupils are explicitly taught to use the bar model as a way to represent a division in a ratio problem</i>	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> • Show me a set of objects that demonstrates the ratio 3:2. And another, and another ... • Convince me that the ratio 120mm:0.3m is equivalent to 2:5 • Always / Sometimes / Never: the smaller number comes first when writing a ratio • Using Cuisenaire rods: If the red rod is 1, explain why d (dark green) is 3. Can you say the value for all the rods? (w, r, g, p, y, d, b, t, B, o). Extend this understanding of proportion by changing the unit rod e.g. if $r = 1$, $p = ?$; $b = ?$; $o + 2B = ?$ If $B = 1$; $y = ?$ $3y = ?$; $o = ?$ $o + p = ?$ If $o + r = 6/7$; $t = ?$ 	KM: Division in a ratio and checking spreadsheet KM: Maths to Infinity: FDP RP KM: Stick on the Maths: Ratio and proportion NRICH: Toad in the hole NRICH: Mixing lemonade NRICH: Food chains NRICH: Tray bake	<ul style="list-style-type: none"> • Some pupils may think that a:b always means part:part • Some pupils may try to simplify a ratio without first ensuring that the units of each part are the same • Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts 	

Key concepts (GCSE subject content statements)

The Big Picture: [Algebra progression map](#)

- generate terms of a sequence from a term-to-term rule

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Possible themes		Possible key learning points	
<ul style="list-style-type: none"> • Investigate number patterns • Explore number sequences • Explore sequences 		<ul style="list-style-type: none"> • Recognise simple arithmetic progressions • Use a term-to-term rule to generate a linear sequence • Use a term-to-term rule to generate a non-linear sequence 	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> • Know the vocabulary of sequences • Find the next term in a linear sequence • Find a missing term in a linear sequence • Generate a linear sequence from its description <p>Bring on the Maths+: Moving on up! Number and Place Value: #4, #5</p>	Pattern Sequence Linear Term Term-to-term rule Ascending Descending	'Term-to-term rule' is the only new vocabulary for this unit. Position-to-term rule, and the use of the nth term, are not developed until later stages. NRICH: Go forth and generalise NCETM: Algebra Common approaches <i>All students are taught to describe the term-to-term rule for both numerical and non-numerical sequences</i>	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> • Show me a (non-)linear sequence. And another. And another. • What's the same, what's different: 2, 5, 8, 11, 14, ... and 4, 7, 10, 13, 16, ...? • Create a (non-linear/linear) sequence with a 3rd term of '7' • Always/ Sometimes /Never: The 10th term of is double the 5th term of the (linear) sequence • Kenny thinks that the 20th term of the sequence 5, 9, 13, 17, 21, ... will be 105. Do you agree with Kenny? Explain your answer. 	KM: Maths to Infinity: Sequences KM: Growing patterns NRICH: Shifting times tables NRICH: Odds and evens and more evens	<ul style="list-style-type: none"> • When describing a number sequence some students may not appreciate the fact that the starting number is required as well as a term-to-term rule • Some pupils may think that all sequences are ascending • Some pupils may think the (2n)th term of a sequence is double the nth term of a (linear) sequence 	

Key concepts (GCSE subject content statements) <ul style="list-style-type: none"> • use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.) • use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate • change freely between related standard units (e.g. time, length, area, volume/capacity, mass) in numerical contexts • measure line segments and angles in geometric figures 	The Big Picture: Measurement and mensuration progression map
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Possible themes	Possible key learning points
<ul style="list-style-type: none"> • Measure accurately • Convert between measures • Solve problems involving measurement 	<ul style="list-style-type: none"> • Use a ruler to accurately measure line segments to the nearest millimetre • Use a protractor to accurately measure angles to the nearest degree • Convert fluently between metric units of length • Convert fluently between metric units of mass • Convert fluently between metric units of volume / capacity • Convert fluently between units of time • Convert fluently between units of money

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Convert between metric units • Use decimal notation up to three decimal places when converting metric units • Convert between common Imperial units; e.g. feet and inches, pounds and ounces, pints and gallons • Convert between units of time • Use 12- and 24-hour clocks, both analogue and digital <p>Bring on the Maths+: Moving on up! Measures: #3</p>	Length, distance Mass, weight Volume Capacity Metre, centimetre, millimetre Tonne, kilogram, gram, milligram Litre, millilitre Hour, minute, second Inch, foot, yard Pound, ounce Pint, gallon Line segment Notation Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml Abbreviations of units in the Imperial system: lb, oz	Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement. The prefix 'centi-' means one hundredth, and the prefix 'milli-' means one thousandth. These words are of Latin origin. The prefix 'kilo-' means one thousand. This is Greek in origin. Classify/Estimate angle first NCETM: Glossary Common approaches <i>Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)</i>

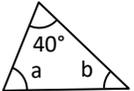
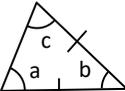
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Show me another way of describing 2.5km. And another. And another. • Show me another way of describing 3.4 litres. And another. And another. • Show me another way of describing 3.7kg. And another. And another. • Kenny thinks that 14:30 is the same time as 2.30 p.m. Do you agree with Kenny? Explain your answer. • What's the same, what's different: 2 hours 30 minutes, 2.5 hours, 2½ hours and 2 hours 20 minutes? 	KM: Sorting units KM: Another length KM: Measuring space KM: Another capacity KM: Stick on the Maths: Units NRICH: Temperature	<ul style="list-style-type: none"> • Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point • Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems • Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock • Some pupils may use the wrong scale of a protractor. For example, they measure an obtuse angle as 60° rather than 120°.

Key concepts (GCSE subject content statements)

The Big Picture: [Position and direction progression map](#)

- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles

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Possible themes		Possible key learning points	
<ul style="list-style-type: none"> • Investigate angles <p>Bring on the Maths+: Moving on up! Properties of shapes: #5</p>		<ul style="list-style-type: none"> • Recognise and solve problems using vertically opposite angles • Recognise and solve problems using angles at a point • Recognise and solve problems using angles at a point on a line 	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> • Identify angles that meet at a point • Identify angles that meet at a point on a line • Identify vertically opposite angles • Know that vertically opposite angles are equal 	Angle Degrees Right angle Acute angle Obtuse angle Reflex angle Protractor Vertically opposite Geometry, geometrical Notation Right angle notation Arc notation for all other angles The degree symbol ($^{\circ}$)	It is important to make the connection between the total of the angles in a triangle and the sum of angles on a straight line by encouraging pupils to draw any triangle, rip off the corners of triangles and fitting them together on a straight line. However, this is not a proof and this needs to be revisited in Stage 8 using alternate angles to prove the sum is always 180° . The word 'isosceles' means 'equal legs'. What do you have at the bottom of equal legs? Equal ankles! NCETM: Glossary Common approaches <i>Teachers convince pupils that the sum of the angles in a triangle is 180° by ripping the corners of triangles and fitting them together on a straight line.</i>	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> • Show me possible values for a and b. And another. And another. • Convince me that the angles in a triangle total 180° • Convince me that the angles in a quadrilateral total 360° • What's the same, what's different: Vertically opposite angles, angles at a point, angles on a straight line and angles in a triangle? • Kenny thinks that a triangle cannot have two obtuse angles. Do you agree? Explain your answer. • Jenny thinks that the largest angle in a triangle is a right angle? Do you agree? Explain your thinking. 	KM: Maths to Infinity: Lines and angles KM: Stick on the Maths: Angles NRICH: Triangle problem NRICH: Square problem NRICH: Two triangle problem	<ul style="list-style-type: none"> • Some pupils may think it's the 'base' angles of an isosceles that are always equal. For example, they may think that $a = b$ rather than $a = c$.  <ul style="list-style-type: none"> • Some pupils may make conceptual mistakes when adding and subtracting mentally. For example, they may see that one of two angles on a straight line is 127° and quickly respond that the other angle must be 63°. 	

Key concepts (GCSE subject content statements)

The Big Picture: [Fractions, decimals and percentages progression map](#)

- apply the four operations, including formal written methods, to simple fractions (proper and improper), and mixed numbers
- interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively
- compare two quantities using percentages
- solve problems involving percentage change, including percentage increase/decrease

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Possible themes	Possible key learning points	
<ul style="list-style-type: none"> • Calculate with fractions • Calculate with percentages 	<ul style="list-style-type: none"> • Add proper and improper fractions • Add mixed numbers • Subtract proper and improper fractions • Subtract mixed numbers • Multiply proper and improper fractions • Multiply mixed numbers • Divide a proper fraction by a proper fraction • Divide improper fractions 	<ul style="list-style-type: none"> • Divide a mixed number by a proper fraction/mixed number • Identify the multiplier for a percentage increase or decrease • Use calculators to find a percentage of an amount using multiplicative methods • Use calculators to increase and decrease an amount by a percentage using multiplicative methods • Compare two quantities using percentages • Know that percentage change = actual change ÷ original amount • Calculate the percentage change in a given situation, including percentage increase / decrease
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Add and subtract fractions with different denominators • Add and subtract mixed numbers with different denominators • Multiply a proper fraction by a proper fraction • Divide a proper fraction by a whole number • Simplify the answer to a calculation when appropriate • Use non-calculator methods to find a percentage of an amount • Convert between fractions, decimals and percentages <p>Bring on the Maths*: Moving on up! Fractions, decimals & percentages: #3, #4, #5 Ratio and proportion: #2</p>	<p>Mixed number Equivalent fraction Simplify, cancel, lowest terms Proper fraction, improper fraction, top-heavy fraction, vulgar fraction Percent, percentage Multiplier Increase, decrease</p> <p>Notation Mixed number notation Horizontal / diagonal bar for fractions</p>	<p>It is important that pupils are clear that the methods for addition and subtraction of fractions are different to the methods for multiplication and subtraction. A fraction wall is useful to help visualise and re-present the calculations. NCETM: The Bar Model, Teaching fractions, Fractions videos NCETM: Glossary</p> <p>Common approaches <i>When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy</i> <i>When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy</i> <i>Teachers use the horizontal fraction bar notation at all times</i></p>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Show me a proper (improper) fraction. And another. And another. • Show me a mixed number fraction. And another. And another. • Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain your answer. • Benny thinks that you can only divide fractions if they have the same common denominator. Do you agree with Benny? Explain. • Kenny thinks that $\frac{6}{10} \div \frac{3}{2} = \frac{2}{5}$. Do you agree with Kenny? Explain. • Always/Sometimes/Never: To reverse an increase of x%, you decrease by x% • Lenny calculates the % increase of £6 to £8 as 25%. Do you agree with Lenny? Explain your answer. 	<p>KM: Stick on the Maths: Percentage increases and decreases KM: Maths to Infinity: FDPRP KM: Percentage methods KM: Mixed numbers: mixed approaches NRICH: Would you rather? NRICH: Keep it simple NRICH: Egyptian fractions NRICH: The greedy algorithm NRICH: Fractions jigsaw NRICH: Countdown fractions</p> <p>Learning review KM: 7M4 BAM Task, 7M5 BAM Task</p>	<ul style="list-style-type: none"> • Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional part of mixed numbers when adding/subtracting mixed numbers, e.g. $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$ • Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators. • Some pupils may think the multiplier for, say, a 20% decrease is 0.2 rather than 0.8 • Some pupils may think that percentage change = actual change ÷ new amount

Key concepts (GCSE subject content statements)

The Big Picture: [Algebra progression map](#)

- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)
- solve linear equations in one unknown algebraically

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Possible themes	Possible key learning points																	
<ul style="list-style-type: none"> • Explore way of solving equations • Solve two-step equations • Solve three-step equations 	<ul style="list-style-type: none"> • Solve one-step equations when the solution is a positive integer or fraction • Solve two-step equations when the solution is a positive integer or fraction • Solve three-step equations when the solution is a positive integer or fraction • Solve multi-step equations including the use of brackets when the solution is a positive integer or fraction • Solve equations when the solution is an integer or fraction 																	
Prerequisites	Mathematical language	Pedagogical notes																
<ul style="list-style-type: none"> • Know the basic rules of algebraic notation • Express missing number problems algebraically • Solve missing number problems expressed algebraically <p>Bring on the Maths*: Moving on up! Algebra: #2</p>	<p>Algebra, algebraic, algebraically Unknown Equation Operation Solve Solution Brackets Symbol Substitute</p> <p>Notation The lower case and upper case of a letter should not be used interchangeably when worked with algebra Juxtaposition is used in place of 'x'. 2a is used rather than a2. Division is written as a fraction</p>	<p>This unit focuses on solving linear equations with unknowns on one side. Although linear equations with the unknown on both sides are addressed in Stage 8, pupils should be encouraged to think how to solve these equations by exploring the equivalent family of equations such as if $2x = 8$ then $2x + 2 = 10$, $2x - 3 = 5$, $3x = x + 8$, $3x + 2 = x + 10$, etc.</p> <p>Encourage pupils to re-present the equations such as $2x + 8 = 23$ using the Bar Model.</p> <table border="1" data-bbox="1910 627 2085 676"> <tr><td>x</td><td>x</td><td>8</td></tr> <tr><td colspan="3">23</td></tr> </table> <table border="1" data-bbox="1910 683 2027 732"> <tr><td>x</td><td>x</td></tr> <tr><td colspan="2">15</td></tr> </table> <table border="1" data-bbox="1910 738 1971 788"> <tr><td>x</td></tr> <tr><td>7.5</td></tr> </table> <p>NCETM: The Bar Model NCETM: Algebra, NCETM: Glossary</p> <p>Common approaches <i>Pupils could explore solving equations by applying inverse operations, but the expectation is that all pupils should solve by balancing:</i></p> $\begin{array}{rcl} 2x + 8 & = & 23 \\ -8 & & -8 \\ \hline 2x & = & 15 \\ \div 2 & & 2 \\ \hline x & = & 7.5 \text{ (or } 15/2) \end{array}$ <p><i>Pupils are expected to multiply out the brackets before solving an equation involving brackets. This makes the connection with two step equations such as $2x + 6 = 22$</i></p>	x	x	8	23			x	x	15		x	7.5				
x	x	8																
23																		
x	x																	
15																		
x																		
7.5																		
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions																
<ul style="list-style-type: none"> • Show me an (one-step, two-step) equation with a solution of 14 (positive, fractional solution). And another. And another ... • Kenny thinks if $6x = 3$ then $x = 2$. Do you agree with Kenny? Explain • Jenny and Lenny are solving: $3(x - 2) = 51$. Who is correct? Explain <table border="0" data-bbox="114 1214 629 1396"> <tr> <td style="text-align: center;">Jenny's solution</td> <td style="text-align: center;">Lenny's solution</td> </tr> <tr> <td>$3(x - 2) = 15$</td> <td>$3(x - 2) = 15$</td> </tr> <tr> <td>$\div 3 \quad \div 3$</td> <td><i>Multiplying out brackets</i></td> </tr> <tr> <td>$x - 2 = 5$</td> <td>$3x - 6 = 15$</td> </tr> <tr> <td>$\div 2 \quad \div 2$</td> <td>$+2 \quad +2$</td> </tr> <tr> <td>$x = 7$</td> <td>$3x = 21$</td> </tr> <tr> <td></td> <td>$\div 3 \quad \div 3$</td> </tr> <tr> <td></td> <td>$x = 7$</td> </tr> </table>	Jenny's solution	Lenny's solution	$3(x - 2) = 15$	$3(x - 2) = 15$	$\div 3 \quad \div 3$	<i>Multiplying out brackets</i>	$x - 2 = 5$	$3x - 6 = 15$	$\div 2 \quad \div 2$	$+2 \quad +2$	$x = 7$	$3x = 21$		$\div 3 \quad \div 3$		$x = 7$	<p>KM: Balancing: Act I KM: Balancing: Act II KM: Balancing: Act III KM: Spiders and snakes. The example is for an unknown on both sides but the same idea can be used. NRICH: Inspector Remorse NRICH: Quince, quonce, quance NRICH: Weighing the baby</p> <p>Learning review KM: 7M10 BAM Task</p>	<ul style="list-style-type: none"> • Some pupils may think that equations always need to be presented in the form $ax + b = c$ rather than $c = ax + b$. • Some pupils may think that the solution to an equation is always positive and/or a whole number. • Some pupils may get the use the inverse operations in the wrong order, for example, to solve $2x + 18 = 38$ the pupils divide by 2 first and then subtract 18.
Jenny's solution	Lenny's solution																	
$3(x - 2) = 15$	$3(x - 2) = 15$																	
$\div 3 \quad \div 3$	<i>Multiplying out brackets</i>																	
$x - 2 = 5$	$3x - 6 = 15$																	
$\div 2 \quad \div 2$	$+2 \quad +2$																	
$x = 7$	$3x = 21$																	
	$\div 3 \quad \div 3$																	
	$x = 7$																	

Key concepts (GCSE subject content statements)

The Big Picture: [Measurement and mensuration progression map](#)

- use standard units of measure and related concepts (length, area, volume/capacity)
- calculate perimeters of 2D shapes
- know and apply formulae to calculate area of triangles, parallelograms, trapezia
- *calculate surface area of cuboids*
- know and apply formulae to calculate volume of cuboids
- understand and use standard mathematical formulae

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Possible themes		Possible key learning points	
<ul style="list-style-type: none"> • Develop knowledge of area • Investigate surface area • Explore volume 		<ul style="list-style-type: none"> • Calculate perimeters of 2D shapes • Use and apply the formula to calculate the area of triangles • Use and apply the formula to calculate the area of trapezia • Use and apply the formula to calculate the volume of cuboids • Find the surface area of cuboids (including cubes) 	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> • Understand the meaning of area, perimeter, volume and capacity • Know how to calculate areas of rectangles, parallelograms and triangles using the standard formulae • Know that the area of a triangle is given by the formula $\text{area} = \frac{1}{2} \times \text{base} \times \text{height} = \text{base} \times \text{height} \div 2 = \frac{bh}{2}$ • Know appropriate metric units for measuring area and volume <p>Bring on the Maths⁺: Moving on up! Measures: #4, #5, #6</p>	Perimeter, area, volume, capacity, surface area Square, rectangle, parallelogram, triangle, trapezium (trapezia) Polygon Cube, cuboid Square millimetre, square centimetre, square metre, square kilometre Cubic centimetre, centimetre cube Formula, formulae Length, breadth, depth, height, width Notation Abbreviations of units in the metric system: km, m, cm, mm, mm ² , cm ² , m ² , km ² , mm ³ , cm ³ , km ³	Ensure that pupils make connections with the area and volume work in Stage 6 and below, in particular the importance of the perpendicular height. NCETM: Glossary Common approaches <i>Pupils have already derived the formula for the area of a parallelogram. They use this to derive the formula for the area of a trapezium as $\frac{(a+b)h}{2}$ by copying and rotating a trapezium as shown above.</i>  <i>Pupils use the area of a triangle as given by the formula $\text{area} = \frac{bh}{2}$.</i> Every classroom has a set of area posters on the wall.	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> • Always / Sometimes / Never: The value of the volume of a cuboid is greater than the value of the surface area • Convince me that the area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height} = \text{base} \times \text{height} \div 2 = \frac{bh}{2}$ • (Given a right-angled trapezium with base labelled 8 cm, height 5 cm, top 6 cm) Kenny uses the formula for the area of a trapezium and Benny splits the shape into a rectangle and a triangle. What would you do? Why? 	KM: Perimeter KM: Triangles KM: Equable shapes (for both 2D and 3D shapes) KM: Triangle takeaway KM: Surface area KM: Class of rice KM: Stick on the Maths: Area and Volume KM: Maths to Infinity: Area and Volume NRICH: Can They Be Equal? Learning review KM: 7M12 BAM Task	<ul style="list-style-type: none"> • Some pupils may use the sloping height when finding the areas of parallelograms, triangles and trapezia • Some pupils may think that the area of a triangle is found using $\text{area} = \text{base} \times \text{height}$ • Some pupils may think that you multiply all the numbers to find the area of a shape • Some pupils may confuse the concepts of surface area and volume • Some pupils may only find the area of the three 'distinct' faces when finding surface area 	

Key concepts (GCSE subject content statements)

The Big Picture: [Position and direction progression map](#)

- work with coordinates in all four quadrants
- understand and use lines parallel to the axes, $y = x$ and $y = -x$
- solve geometrical problems on coordinate axes
- identify, describe and construct congruent shapes including on coordinate axes, by considering rotation, reflection and translation
- describe translations as 2D vectors

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Possible themes		Possible key learning points
<ul style="list-style-type: none"> • Explore lines on the coordinate grid • Use transformations to move shapes • Describe transformations 		<ul style="list-style-type: none"> • Solve geometrical problems on coordinate axes • Write the equation of a line parallel to the x-axis or the y-axis • Identify and draw the lines $y = x$ and $y = -x$ • Construct and describe reflections in horizontal, vertical and diagonal mirror lines (45° from horizontal) • Describe a translation as a 2D vector • Construct and describe rotations using a given angle, direction and centre of rotation • Solve problems involving rotations, reflections and translations
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Work with coordinates in all four quadrants • Carry out a reflection in a given vertical or horizontal mirror line • Carry out a translation <p>Bring on the Maths+: Moving on up! Position and direction: #1, #2</p>	(Cartesian) coordinates Axis, axes, x-axis, y-axis Origin Quadrant Translation, Reflection, Rotation Transformation Object, Image Congruent, congruence Mirror line Vector Centre of rotation Notation Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y) Vector notation $\begin{pmatrix} a \\ b \end{pmatrix}$ where a = movement right and b = movement up	Pupils should be able to use a centre of rotation that is outside, inside, or on the edge of the object Pupils should be encouraged to see the line $x = a$ as the complete (and infinite) set of points such that the x-coordinate is a. The French mathematician Rene Descartes introduced Cartesian coordinates in the 17 th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling. NCETM: Glossary Common approaches <i>Pupils use ICT to explore these transformations</i> <i>Teachers do not use the phrase ‘along the corridor and up the stairs’ as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is ‘x is a cross, wise up!’</i> <i>Teachers use the language ‘negative number’, and not ‘minus number’, to avoid future confusion with calculations.</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Always / Sometimes / Never: The centre of rotation is in the centre of the object • Convince me that $y = 0$ is the x-axis • Always / Sometimes / Never: The line $x = a$ is parallel to the x-axis 	KM: Lines KM: Moving house KM: Transformations: Bop It? KM: Dynamic Autograph files: Reflection , Rotation , Translation KM: Autograph transformations KM: Stick on the Maths SSM7: Transformations NRICH: Transformation Game Learning review KM: 7M11 BAM Task	<ul style="list-style-type: none"> • Some pupils will wrestle with the idea that a line $x = a$ is parallel to the y-axis • When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes. • When reflecting a shape in a diagonal mirror line some students may draw a translation • Some pupils may think that the centre of rotation is always in the centre of the shape • Some pupils will confuse the order of x- and y-coordinates • When constructing axes, some pupils may not realise the importance of equal divisions on the axes

Key concepts (GCSE subject content statements)

The Big Picture: [Statistics progression map](#)

- interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data and know their appropriate use

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Possible themes		Possible key learning points
<ul style="list-style-type: none"> • Explore types of data • Construct and interpret graphs • Select appropriate graphs and charts 		<ul style="list-style-type: none"> • Interpret and construct frequency tables • Construct and interpret bar charts and know their appropriate use • Construct and interpret comparative bar charts • Construct and interpret pie charts and know their appropriate use • Construct and interpret vertical line charts • Choose appropriate graphs or charts to represent data
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Construct and interpret a pictogram • Construct and interpret a bar chart • Construct and interpret a line graph • Understand that pie charts are used to show proportions • Use a template to construct a pie chart by scaling frequencies <p>Bring on the Maths*: Moving on up! Statistics: #1, #2, #3</p>	Data, Categorical data, Discrete data Pictogram, Symbol, Key Frequency Table, Frequency table Tally Bar chart Time graph, Time series Bar-line graph, Vertical line chart Scale, Graph Axis, axes Line graph Pie chart Sector Angle Maximum, minimum Notation When tallying, groups of five are created by striking through each group of four	In stage 6 pupils constructed pie charts when the total of frequencies is a factor of 360. More complex cases can now be introduced. Much of the content of this unit has been covered previously in different stages. This is an opportunity to bring together the full range of skills encountered up to this point, and to develop a more refined understanding of usage and vocabulary. William Playfair, a Scottish engineer and economist, introduced the bar chart and line graph in 1786. He also introduced the pie chart in 1801. NCETM: Glossary Common approaches <i>Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency and not using percentages. The angle for the first section is measured from a vertical radius. Subsequent sections are measured using the boundary line of the previous section.</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> • Show me a pie chart representing the following information: Blue (30%), Red (50%), Yellow (the rest). And another. And another. • Always / Sometimes / Never: Bar charts are vertical • Always / Sometimes / Never: Bar charts, pie charts, pictograms and vertical line charts can be used to represent any data • Kenny says 'If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.' Do you agree with Kenny? Explain your answer. 	KM: Constructing pie charts KM: Maths to Infinity: Averages, Charts and Tables NRICH: Picturing the World NRICH: Charting Success	<ul style="list-style-type: none"> • Some pupils may think that the lines on a line graph are always meaningful • Some pupils may think that each square on the grid used represents one unit • Some pupils may confuse the fact that the sections of the pie chart total 100% and 360° • Some pupils may not leave gaps between the bars of a bar chart

Key concepts (GCSE subject content statements)

The Big Picture: [Statistics progression map](#)

- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean and mode) and spread (range)

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Possible themes		Possible key learning points	
<ul style="list-style-type: none"> • Investigate averages • Explore ways of summarising data • Analyse and compare sets of data 		<ul style="list-style-type: none"> • Find the mode of set of data • Find the median of a set of data including when there are an even number of numbers in the data set • Calculate the mean from a frequency table • Find the mode from a frequency table • Find the median from a frequency table • Calculate and understand the range as a measure of spread (or consistency) • Analyse and compare sets of data, appreciating the limitations of different statistics (mean, median, mode, range) 	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> • Understand the meaning of 'average' as a typicality (or location) • Calculate the mean of a set of data <p>Bring on the Maths*: Moving on up! Statistics: #4</p>	Average Spread Consistency Mean Median Mode Range Measure Data Statistic Statistics Approximate Round	The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean'). NCETM: Glossary Common approaches <i>Every classroom has a set of statistics posters on the wall</i> <i>Always use brackets when writing out the calculation for a mean, e.g. $(2 + 3 + 4 + 5) \div 4 = 14 \div 4 = 3.5$</i>	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> • Show me a set of data with a mean (mode, median, range) of 5. • Always / Sometimes / Never: The mean is greater than the mode for a set of data • Always / Sometimes / Never: The mean is greater than the median for a set of data • Convince me that a set of data could have more than one mode. • What's the same and what's different: mean, mode, median, range? 	KM: Maths to Infinity: Averages KM: Maths to Infinity: Averages, Charts and Tables KM: Stick on the Maths HD4: Averages NRICH: M, M and M NRICH: The Wisdom of the Crowd	<ul style="list-style-type: none"> • If using a calculator some pupils may not use the '=' symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as $2 + 3 + 4 + 5 \div 4 = 10.25$. • Some pupils may think that the range is a type of average • Some pupils may think that a set of data with an even number of items has two values for the median, e.g. 2, 4, 5, 6, 7, 8 has a median of 5 and 6 rather than 5.5 • Some pupils may not write the data in order before finding the median. 	