Oakfield Lodge School Nurture Mathematics Scheme of Learning 2021-2022

Unit	*Lessons*	*Key 'Build a Mathematician' (BAM) Indicators*	*Essential knowledge*
Numbers and the number system	12	• Multiply and divide numbers with up to three decimal places by 10, 100, and 1000 • Use long division to divide numbers up to four digits by a two-digit number • Use simple formulae expressed in words • Generate and describe linear number sequences • Use simple ratio to compare quantities • Write a fraction in its lowest terms by cancelling common factors • Add and subtract fractions and mixed numbers with different denominators • Multiply pairs of fractions in simple cases • Find percentages of quantities • Solve missing angle problems involving triangles, quadrilaterals, angles at a point and angles on a straight line • Calculate the volume of cubes and cuboids • Use coordinates in all four quadrants • Calculate and interpret the mean as an average of a set of discrete data Stage 6 BAM Progress Tracker Sheet	• Know percentage and decimal equivalents for fractions with a denominator of 2, 3, 4, 5, 8 and 10 • Know the rough equivalence between miles and kilometres • Know that vertically opposite angles are equal • Know that the area of a triangle = base × height ÷ 2 • Know that the area of a parallelogram = base × height • Know that volume is measured in cubes • Know the names of parts of a circle • Know that the diameter of a circle is twice the radius • Know the conventions for a 2D coordinate grid • Know that mean = sum of data ÷ number of pieces of data
Checking, approximating and estimating	7		
Calculating	11		
Calculating: division	7		
Visualising and constructing	8		
Investigating properties of shapes	8		
Algebraic proficiency: using formulae	4		
Exploring fractions, decimals and percentages	8		
Proportional reasoning	6		
Pattern sniffing	5		
Measuring space	6		
Investigating angles	4		
Calculating fractions, decimals and percentages	12		
Solving equations and inequalities	4		
Calculating space	8		
Mathematical movement	4		
Presentation of data	4		
Measuring data	4		
Total:	122		

Maths Calendar

Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12	Week 13
Numbers and the number system 6M1 BAM				Checking, approximating etc		Calculating 6M2 BAM			Calculating: division		Visualising and constructing	

Week 14	Week 15	Week 16	Week 17	Week 18	Week 19	Week 20	Week 21	Week 22	Week 23	Week 24	Week 25	Week 26
Assessment and enrichment		Investigating properties of shapes		Formulae 6M3 BAM		Exploring FDP 6M6 BAM		Proportional reasoning 6M5 BAM		Pattern sniffing 6M4 BAM	Measuring space	

Week 27	Week 28	Week 29	Week 30	Week 31	Week 32	Week 33	Week 34	Week 35	Week 36	Week 37	Week 38	Week 39
Assessment	Angles 6M10 BAM	Calculating fractions, decimals and percentages 6M7 BAM, 6M8 BAM, 6M9 BAM			Solving equations	Calculating space 6M11 BAM		Movement 6M12 BAM		Presenting data	Measuring data 6M13 BAM	Assessment

Numbers and the number system — 12 lessons

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Number and Place Value progression map

- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- use negative numbers in context, and calculate intervals across zero
- identify common factors, common multiples and prime numbers

Return to overview

Possible themes

- Understand and use decimals with up to three decimal places
- Work with numbers up to ten million
- Explore the use of negative numbers
- Develop understanding of factors and multiples
- Investigate prime numbers

Bring on the Maths⁺: Moving on up!
Number and Place Value: #3, #6
Calculating: #2

Possible key learning points

- Write and read numbers up to and including 10 000 000
- Compare and order numbers up to and including 10 000 000
- Multiply numbers by 10
- Multiply numbers by 100
- Multiply numbers by 1000
- Divide numbers by 10
- Divide numbers by 100
- Divide numbers by 1000
- Understand and use negative numbers when working in context, such as temperature
- Calculate intervals across zero
- Find common multiples of two numbers
- Find common factors of two numbers

Prerequisites

- Understand and use place value in numbers with up to seven digits
- Multiply and divide whole numbers by 10, 100, 1000
- Multiply and divide numbers with one decimal place by 10, 100, 1000
- Know the meaning of 'factor' and 'multiple' and 'prime'

Bring on the Maths⁺: Moving on up!
Number and Place Value: #1
Solving problems: #3

Mathematical language

Place value
Digit
Negative number
(Common) multiple
(Common) factor
Divisible
Prime number, Composite number

Pedagogical notes

Zero is neither positive nor negative.
When multiplying and dividing by powers of ten, the decimal point is fixed and it is the digits that move.
Ensure that pupils can deal with large numbers that include zeros in the HTh and/or H column (e.g. 43 006 619)
NCETM: Glossary

Common approaches
The following definition of a prime number is used in order to minimise confusion about 1: A prime number is a number with exactly two factors. Every classroom has a set of number classification posters, a place chart and a negative number washing line on the wall.

Reasoning opportunities and probing questions

- Convince me that 109 is a prime number
- Jenny writes 2.54 × 10 = 25.4. Kenny writes 2.54 × 10 = 25.40. who do you agree with? Explain why.
- Look at this number (24 054 028). Show me another number (with 4, 5, 6, 7 digits) that includes a 5 with the same value. And another. And another …

NCETM: Place Value Reasoning

Suggested activities

KM: Maths to Infinity: Directed numbers
KM: Reading and writing
KM: Extend the idea of Eratosthenes' sieve to a 12 by 12 grid
KM: Exploring primes activities: Artistic Eratosthenes sieve
KM: Use Powers of ten to demonstrate connections.
NRICH: Factor-multiple chains
NRICH: The Moons of Vuvv
NRICH: Round and round the circle
NRICH: Counting cogs

Learning review
KM: 6M1 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils can confuse the language of large (and small) numbers since the prefix 'milli- means 'one thousandth' (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand.
- Some pupils may not realise that degrees (°) and degrees Celsius (°C) are two different and distinct units of measurement
- Some pupils may think that 1 is a prime number

Checking, approximating and estimating — *7 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Number and Place Value progression map

- solve problems which require answers to be rounded to specified degrees of accuracy
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- round any whole number to a required degree of accuracy

Return to overview

Possible themes

- Explore ways of approximating numbers
- Explore ways of checking answers

Possible key learning points

- Round a number to the nearest 10
- Round a number to the nearest 100
- Round a number to the nearest 1000
- Round a number to the nearest whole number
- Round a number to the nearest 1 decimal place
- Round a number to the nearest 2 decimal places
- Understand estimating as the process of finding a rough value of an answer or calculation

Prerequisites

- Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000 or 100 000
- Approximate any number with one or two decimal places by rounding to the nearest whole number
- Approximate any number with two decimal places by rounding to the one decimal place
- Estimate addition (subtraction) calculations with up to four digits

Mathematical language

Approximate (noun and verb)
Round
Decimal place
Check
Solution
Answer
Estimate (noun and verb)
Order of magnitude
Accurate
Accuracy

Notation
The approximately equal symbol (≈)

Pedagogical notes

Pupils should use numbers up to 10 000 000 in this unit.
Pupils should be able to round to other specified degrees of accuracy, but not to a specified number of significant figures, which is introduced in Stage 7.
Also see big pictures: Calculation progression map and Fractions, decimals and percentages progression map
NCETM: Glossary

Common approaches
All pupils are taught to visualise rounding through the use a number line

Reasoning opportunities and probing questions

- Convince me that 67 rounds to 60 to the nearest 20
- Convince me that 1 579 234 rounds to 2 million to the nearest million
- Jenny writes 1359 ÷ 18 ≈ 7.55. Comment on Jenny's approximation.
- Lenny writes 2.74 × 13 ≈ 26. Do you agree with Lenny? Explain your answer.

NCETM: Place Value Reasoning

Suggested activities

KM: Checking, approximating, estimating
KM: Stick on the Maths CALC6: Checking results
KM: Maths to Infinity Rounding
KM: Alfred and the prize money
NRICH: Four Go
NCETM: Activity A(i)
NCETM: Activity G

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may truncate instead of round
- When checking the order of magnitude of a division calculation some pupils may apply incorrect reasoning about the effect of increasing the divisor by a factor of 10, thinking that it also makes the solution greater by a factor of 10; e.g. 1400 ÷ 20: 1400 ÷ 2 = 700 so 1400 ÷ 20 = 7000.
- Some pupils may round down at the half way point, rather than round up.

Calculating — 11 lessons

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Calculation progression map

- perform mental calculations, including with mixed operations and large numbers
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- solve problems involving addition, subtraction and multiplication
- use their knowledge of the order of operations to carry out calculations

Return to overview

Possible themes

- Develop mental calculation skills
- Extend written methods of multiplication
- Know and use the order of operations
- Solve problems involving addition, subtraction and multiplication

Bring on the Maths⁺: Moving on up!
Calculating: #4
Fractions, decimals & percentages: #6
Solving problems: #2

Possible key learning points

- Carry out addition calculations mentally involving numbers up to 4 digits
- Carry out subtraction calculations mentally involving numbers up to 4 digits
- Solve addition and subtraction multi-step problems in context
- Multiply a two or three-digit numbers by a two-digit number
- Multiply a four-digit number by a two-digit number using long multiplication
- Carry out calculations involving a mixture of multiplication and division
- Carry out calculations involving mixture of addition and subtraction
- Carry out calculations involving mixture of multiplication and addition/subtraction
- Carry out calculations involving mixture of division and addition/subtraction
- Solve multi-step problems involving addition, subtraction and/or multiplication
- Check the order of magnitude of the solution to a calculation, including decimals

Prerequisites

- Recall multiplication facts for multiplication tables up to 12 × 12
- Recall division facts for multiplication tables up to 12 × 12
- Understand the commutativity of multiplication and addition
- Multiply a three-digit number by a two-digit number using short multiplication
- Use column addition and subtraction for numbers with more than four digits

Bring on the Maths⁺: Moving on up!
Calculating: #1
Solving problems: #1

Mathematical language

Addition
Subtraction
Sum, Total
Difference, Minus, Less
Column addition
Column subtraction
Operation
Multiply, Multiplication, Times, Product
Commutative
Factor
Short multiplication
Long multiplication
Estimate

Pedagogical notes

Note that if not understood fully, BIDMAS can give the wrong answer to a calculation; e.g. 6 – 2 + 3.
The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.
Use a basic and a scientific calculator to work out 2 + 3 × 5. Why are the answers different?
KM: Progression: Addition and Subtraction, Progression: Multiplication and Division and Calculation overview
NCETM: The Bar Model, Subtraction, Multiplication, Multiplicative reasoning, Glossary

Common approaches
All classrooms display a times table poster with a twist
To avoid confusion with language, all teachers use 'sum' to refer only to the result of an addition.
Teachers say 'complete these calculations' instead of 'complete these sums'
Long multiplication is promoted as the 'most efficient method'.
If any acronym is promoted to help remember the order of operations, then BIDMAS is used to strengthen progression as the I stands for indices.

Reasoning opportunities and probing questions

- Find missing digits in otherwise completed long multiplication calculations
- Convince me that 2472 × 12 = 29664
- Why have you chosen to add (subtract, multiply)?

NCETM: Addition and Subtraction Reasoning
NCETM: Multiplication and Division Reasoning

Suggested activities

KM: Long multiplication template
KM: Maximise, minimise. Adapt ideas to fit learning intentions.
KM: Maths to Infinity: Complements
KM: Maths to Infinity: Multiplying and dividing
NRICH: Become Maths detectives
NRICH: Exploring number patterns you make
NRICH: Reach 100

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may write statements such as 140 - 190 = 50
- When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166.
- The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.

Calculating: division — 7 lessons

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Calculation progression map

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division; interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- use written division methods in cases where the answer has up to two decimal places
- solve problems involving division
- use their knowledge of the order of operations to carry out calculations involving the four operations

Return to overview

Possible themes

- Develop written methods of short division for numbers up to four-digits divided by a one-digit number
- Deal with remainders when carrying out division
- Solve problems involving the four operations

Bring on the Maths⁺: Moving on up!
Calculating: #5

Possible key learning points

- Divide a three-digit number by a two-digit number using a formal written method of division when there is no remainder
- Divide a three-digit number by a two-digit number using a formal written method of when there is a remainder
- Divide a four-digit number by a two-digit number using a formal written method of division when there is no remainder
- Divide a four-digit number by a two-digit number using a formal written method of division when there is a remainder
- Understand how to write the remainder to a division problem as a whole number remainder or as a fraction
- Understand how to interpret remainders to a division problem appropriately for the context
- Solve problems involving division

Prerequisites

- Use knowledge of multiplication tables when dividing
- Know how to use short division

Mathematical language

Commutative
Divide, Division, Divisible
Divisor, Dividend, Quotient, Remainder
Factor
Short division
Long division
Remainder
Operation
Estimate

Notation
Remainders are often abbreviated to ‘r’ or ‘rem’

Pedagogical notes

Long division is a method of division that formalises how to find a remainder at each step. Short division is a compact method that uses pupils’ abilities to find this remainder without the need for a formal written method.
An alternative to both these methods is ‘Galley division’.
KM: Progression: Multiplication and Division and Calculation overview
NCETM: The Bar Model, Division, Glossary

Common approaches
The use of long multiplication is promoted as the ‘most efficient method’.
Short division is promoted as the ‘most efficient method’.
When dealing with remainders in division problems, use the notation ‘r’

Reasoning opportunities and probing questions

- Find missing digits in otherwise completed long / short division calculations
- Show me a calculation that is connected to 147 × 26 = 3822. And another, and another …
- Show me a division calculation that has no remainder. Now show me a division by a two-digit number that has no remainder. And now, a four-digit number divided by a two-digit number that has no remainder. And now, with a remainder of 3 …

NCETM: Multiplication and Division Reasoning

Suggested activities

KM: Dividing (lots)
KM: Maths to Infinity: Multiplying and dividing
KM: Interactive long division
KM: Interactive target boards
KM: Long division development
KM: Maths to Infinity: Multiplication and division foundations
NRICH: Interactive division
NRICH Dicey Operations. Game 6.

Learning review
KM: 6M2 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may write statements such as 12 ÷ 132 = 11
- Formal written methods of addition, subtraction and multiplication work from right to left. Formal division works from left to right.
- When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:

```
        0   10   7    r 5
    ┌──────────────────
  8 │ 3   ⁸6   ⁶1
```

3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been ‘moved across’ and therefore everything that follows has been correctly carried out based on an early misunderstanding.

Visualising and constructing	*8 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Properties of Shape progression map

- draw 2-D shapes using given dimensions and angles
- recognise, describe and build simple 3-D shapes, including making nets

Return to overview

Possible themes

- Construct 2D shapes
- Investigate 3D shapes
- Explore nets of 3D shapes

Bring on the Maths⁺: Moving on up!
Properties of shapes: #1, #2, #3, #4

Possible key learning points

- Draw 2-D shapes given angles
- Draw 2-D shapes given dimensions and angles
- Recognise prisms
- Recognise pyramids
- Classify 3-D shapes including cylinders, cones and spheres
- Build 3-D shapes from nets
- Draw nets of 3-D shapes
- Solve 3-D problems using nets including visualising the edges (vertices) that will meet when folded

Prerequisites

- Know the names of common 2D shapes
- Know the names of common 3D shapes
- Use a protractor to measure and draw angles

Mathematical language

Protractor
Measure
Nearest
Construct
Sketch
Cube, Cuboid, Cylinder, Pyramid, Prism
Net
Edge, Face, Vertex (Vertices)
Visualise

Notation
Dash notation to represent equal lengths in shapes and geometric diagrams
Right angle notation

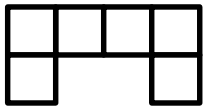
Pedagogical notes

A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid.
A cube is a special case of a cuboid, and a cuboid is a special case of a prism.
Many pupils struggle to sketch 3D shapes. A good strategy for any type of prism is to draw the cross-section (using squares for guidance), and then draw a second identical shape offset from the first. The matching corners can then be joined with straight lines. Some dotted lines (or rubbing out of lines) will be required.
NCETM: Glossary

Common approaches
Every classroom has a set of 3D shape posters on the wall
Models of 3D shapes to be used by all students during this unit of work
All pupils to use 'Polydron' to explore nets of 3D shapes during this unit of work

Reasoning opportunities and probing questions

- Show me an example of a net of a cube. And another. And another …
- What is wrong with this attempt at a net of a cuboid? How can it be changed?



- How many different ways are there to complete these nets?

NCETM: Geometry - Properties of Shapes Reasoning

Suggested activities

KM: Visualising 3D shapes
KM: Tessellating Tess
KM: Fibonacci's disappearing squares
KM: Unravelling dice
KM: Investigate 'Platonic Solids'
NRICH: Making spirals
NRICH: Cut nets
NRICH: Making cuboids

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils will read the wrong way round the scale on a typical semi-circular protractor, therefore using 180° - required angle
- Some pupils may measure from the end of a ruler, rather than the start of the measuring scale
- Some pupils may think that several repeats of a shape in any pattern constitutes a tessellation
- When given a net of a 3D shape some pupils may think that the number of vertices of the 3D shape is found by counting the number of 'corners' on the net

Investigating properties of shapes — *8 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Properties of Shape progression map

- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius

Return to overview

Possible themes

- Investigate properties of 2D shapes
- Investigate angles in polygons
- Understand and use the vocabulary of circles

Bring on the Maths⁺: Moving on up!
Properties of shapes: #5

Possible key learning points

- Classify 2D shapes using given categories; e.g. number of sides, symmetry
- Find unknown angles in a triangle
- Find unknown angles in an isosceles triangle when only one angle is known
- Find unknown angles in a quadrilateral
- Find unknown angles in regular polygons
- Solve problems involving missing angles
- Solve problems involving 2-D shapes
- Know the names and relationships of the parts a circle

Prerequisites

- Know the properties of rectangles
- Know the difference between a regular and an irregular polygon
- Add and subtract numbers up to three digits

Mathematical language

Quadrilateral, Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus, Delta, Arrowhead
Triangle, Scalene, Right-angled, Isosceles, Equilateral
Polygon, Regular, Irregular
Pentagon, Hexagon, Octagon, Decagon, Dodecagon
Circle, Radius, Diameter, Circumference, Centre
Parallel
Diagonal
Angle

Notation
Dash notation to represent equal lengths in shapes and geometric diagrams
Right angle notation

Pedagogical notes

Ensure that pupils do not use the word 'diamond' to describe a kite, or a square that is 45° to the horizontal. 'Diamond' is not the mathematical name of any shape.
A square is a special case of a rectangle. An oblong is a rectangle that is not a square.
A rhombus is a special case of a parallelogram.
All polygons up to 20 sides have names, although many have alternatives based on either Latin or Greek.
Splitting any polygon into triangles (by drawing all diagonals from one vertex) will allow pupils to find the angle sum of the polygon.
NCETM: Glossary

Common approaches
All teachers refer to a 'delta' instead of an 'arrowhead'
Every classroom has a set of triangle posters and quadrilateral posters on the wall

Reasoning opportunities and probing questions

- Convince me that a rhombus is a parallelogram
- Jenny writes that 'Diameter = 2 × Radius'. Kenny writes that 'Radius = 2 × Diameter'. Who is correct?
- What is the same and what is different: a square and a rectangle?

NCETM: Geometry - Properties of Shapes Reasoning

Suggested activities

KM: Investigate 'Tessellations'
KM: Investigate exploring Pi (ratio of the circumference to the diameter)
KM: Shape work: Many of the activities are suitable for this unit.
KM: Dotty activities
KM: Investigating polygons. Tasks one and two.
KM: Special polygons
NRICH: Where Are They?
NRICH: Round a Hexagon
NRICH: Quadrilaterals
KM: 6 point circles, 8 point circles and 12 point circles can be used to support and extend the above idea

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may think that a 'regular' polygon is a 'normal' polygon
- Some pupils may think that all polygons have to be regular
- Some pupils may think that a square is only square if 'horizontal', and even that a 'non-horizontal' square is called a diamond
- The equal angles of an isosceles triangle are not always the 'base angles' as some pupils may think

Exploring fractions, decimals and percentages — *8 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Fractions, decimals and percentages progression map

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $^3/_8$]
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

Return to overview

Possible themes

- Explore the equivalence between fractions
- Use the equivalence between fractions
- Explore the equivalence between fractions, decimals and percentages

Bring on the Maths⁺: Moving on up!
Fractions, decimals & percentages: #1, #2

Possible key learning points

- Use common factors to simplify fractions
- Use common multiples to find equivalent fractions
- Compare and order fractions (fractions < 1)
- Compare and order fractions, including fractions > 1
- Understand a fraction is associated with division
- Work out the decimal equivalents of fifths, eighths and tenths
- Know simple fractions, decimals and percentages equivalences (e.g. 10%, 20%, 25%, 50%, 75%, 100%)
- Find equivalences between fractions, decimals and percentages

Prerequisites

- Understand the concept of a fraction as a proportion
- Understand the concept of equivalent fractions
- Understand the concept of fractions, decimals and percentages being equivalent
- Know that a percentage means 'out of 100'

Mathematical language

Fraction
Improper fraction, Proper fraction, Vulgar fraction, Top-heavy fraction
Percentage
Decimal
Proportion
Simplify
Equivalent
Lowest terms

Notation
Diagonal fraction bar / horizontal fraction bar

Pedagogical notes

Use language carefully to avoid later confusion: when simplifying fractions, the language 'divide by 4' should not be used in place of 'divide the top and bottom by 4'. A fraction can be divided by 4, but that is not the same as cancelling a common factor of the numerator and denominator by dividing them by 4.
NRICH: Teaching fractions with understanding
NCETM: Teaching fractions
NCETM: Glossary

Common approaches
All pupils are made aware that 'per cent' is derived from Latin and means 'out of one hundred'
Teachers use the horizontal fraction bar notation at all times

Reasoning opportunities and probing questions

- Show me another fraction that is equivalent to this one. And another. And another …
- Convince me that 3/8 = 0.375
- If you know that 1/10 = 0.1 = 10%, what else can you work out?
- Jenny is simplifying fractions. She has the fraction 16/64. Jenny says, 'if I cancel out the sixes then 16/64 = 1/4.'. Do you agree with Jenny? Why?

NCETM: Fractions Reasoning

Suggested activities

KM: FDP conversion
KM: Carpets
KM: Fraction and decimal tables
NRICH: Matching fractions
NRICH: Fractions made faster

Learning review
KM: 6M6 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape.
- Pupils may not make the connection that a percentage is a different way of describing a proportion
- Some pupils may think that simplifying a fraction just requires searching for, and removing, a factor of 2 (repeatedly)

Algebraic proficiency: using formulae — *4 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

- use simple formulae
- convert between miles and kilometres

The Big Picture: Algebra progression map

Return to overview

Possible themes

- Use simple formulae written in words
- Create simple formulae written in words
- Work with formulae written algebraically

Bring on the Maths⁺: Moving on up!
Algebra: #1

Possible key learning points

- Use a simple one-step formula written in words
- Use a simple two-step formula written in words
- Use simple formula expressed in symbols
- Convert between miles and kilometres

Prerequisites

- Know the order of operations
- Know the fact that area of rectangle = length × width

Mathematical language

Formula, Formulae
Expression
Variable
Substitute
Symbol
Mile
Kilometre
Metric
Imperial

Notation
When written algebraically a formula should not include any units.

Pedagogical notes

Pupils have already used the written formula 'area of rectangle = length × width'. This can be used here to introduce the use of letters to represent variables; 'A = l × w'. Later in the year pupils will meet other formulae for area and volume and this unit should be used to develop conceptual understanding in readiness for this. Other common examples that could be used include the rough conversion between miles and kilometres, 'kilometres = miles × 1.6'.
NCETM: Algebra
NCETM: Glossary

Common approaches

Reasoning opportunities and probing questions

- Look at this formula. Write down a fact that it tells you. And another. And another …
- Jenny and Kenny are using the formula 'Cost in pounds = 40 + 20 × number of hours' to work out the cost for three hours. Jenny writes down £180. Kenny writes down £100. Who do you agree with? Why?
- Always / Sometimes / Never: The formula T = 4n + 6 results in an odd number.

NCETM: Algebra Reasoning

Suggested activities

KM: Fascinating food
NCETM: Year 6 Algebra. Activities A and D.

Learning review
KM: 6M3 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may apply the order of operations incorrectly when working with two step formulae
- Units must be consistent when using formulae. For example, a mobile phone plan might charge £15 per month plus 5p for every text. The formula 'Monthly cost = 15 + 5 × number of texts is wrong because amounts in both pounds and pence are involved. Monthly cost (in pence) = 1500 + 5 × number of texts is one correct way of writing the formula.
- It is not advisable to abbreviate the formula 'kilometres = miles × 1.6' using letters. 'm' is the normal abbreviation for metres and 'k' can represent £1000. If 'km' is used it could even be interpreted as 'k × m'.

Proportional reasoning	*6 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Ratio and Proportion progression map

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Return to overview

Possible themes

- Solve problems involving scaling
- Explore enlargement
- Solve problems involving sharing and grouping

Bring on the Maths⁺: Moving on up!
Ratio and proportion: #1
Y7 Bring on the Maths
Problem Solving: #1, #2, #3

Possible key learning points

- Solve simple problems involving ratio written in words
- Solve problems involving ratio written in words
- Use a scale factor to solve problems involving similar shapes
- Find the scale factor of similar shapes
- Solve problems involving unequal sharing or grouping problems using fractions
- Solve problems involving unequal sharing or grouping problems using multiples

Prerequisites

- Recall multiplication facts for multiplication tables up to 12 × 12
- Recall division facts for multiplication tables up to 12 × 12
- Find fractions of an amount
- Find multiples of a given number

Mathematical language

Proportion
Ratio
Quantity
Integer
Similar (shapes)
Enlargement
Scale factor
Group
Share
Multiples

Pedagogical notes

Any work on enlargement should only include enlargements using a scale factor. The concept of a centre of enlargement is a future development.
NCETM: The Bar Model
NCETM: Multiplicative reasoning
NCETM: Glossary

Common approaches
All pupils are explicitly taught to use the bar model as a way to represent a problem involving proportional reasoning

Reasoning opportunities and probing questions

- (Given a recipe for 4 people) show me an amount of food that is needed for 8 people, 6 people, 9 people. Show me an amount of food that is needed for a number of people of your choice. And another. And another …
- Convince me that the second shape is an enlargement of the first shape
- Kenny has no sweets. Jenny gives 1/3 of her sweets to Kenny. Jenny now has 18 sweets. Kenny thinks that Jenny had 54 sweets to start with. Kenny is wrong. Explain why.

NCETM: Ratio and Proportion Reasoning

Suggested activities

KM: Proportional reasoning tables
NRICH: Orange Drink
NRICH: Pumpkin Pie Problem
NRICH: Jumping
NCETM: Activity Set A
NCETM: Activity Set B
NCETM: Activity Set C
NCETM: Activity Set D

Learning review
KM: 6M5 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts
- When finding a fraction of an amount some pupils may try to use a rule formed without the necessary understanding. As a result they will muddle the operations, dividing by the numerator and multiplying by the denominator.
- When constructing an enlargement some pupils may only apply the scale factor in one dimension; for example, 'enlarging' a 2 by 4 rectangle by a scale factor of 2 and drawing a 2 by 8 rectangle.

Pattern sniffing	*5 lessons*
Key concepts (Upper Key Stage 2 National Curriculum statements) • generate and describe linear number sequences	**The Big Picture**: Algebra progression map

Return to overview

Possible themes	Possible key learning points
• Explore number sequences **Bring on the Maths+: Moving on up!** Number and Place Value: #4 Number and Place Value: #5	• Recognise and describe a linear sequence • Find the next terms in a linear sequence • Find a missing term in a linear sequence • Generate a linear sequence from its description • Solve problems involving linear sequences

Prerequisites	Mathematical language	Pedagogical notes
• Count forwards and backwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000) • Count forwards and backwards through zero	Pattern Sequence Linear Term Ascending Descending	Pupils have counted forwards and backwards in previous years and units, but this is the first time that the concept of sequences appears specifically. The language 'term-to-term rule' should not be introduced until Stage 7. NCETM: Algebra NCETM: Glossary **Common approaches** *Teachers and pupils refer to numbers less than zero as 'negative' numbers and not 'minus' numbers*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a (ascending/descending) linear sequence. And another. And another. • Kenny thinks that 2, 4, 8, 16, … is a linear example. Do you agree? Explain your answer. • Create a linear sequence with a 3rd term of '8'. • Show me a linear sequence where the rule to get from one term to the next is 'add 3'. And another. And another. NCETM: Algebra Reasoning	KM: Maths to Infinity: Sequences NRICH: Times Tables Shifts NRICH: Domino Sets NCETM: Activity B: Sticky Triangles NCETM: Activity D: Generating Sequences **Learning review** KM: 6M4 BAM Task NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• Some pupils may think linear sequences are only ascending. • Some pupils may think that any sequence that can be described by a rule to get from one term to the next is a linear sequence, e.g. 2, 4, 8, 16, … • Some pupils may not appreciate that both a starting number and a rule to find the next term are required in order to describe a sequence in full.

Measuring space	6 lessons

Key concepts (Upper Key Stage 2 National Curriculum statements) **The Big Picture**: Measurement and mensuration progression map

- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places

Return to overview

Possible themes

- Solve problems involving measurement

Bring on the Maths⁺: Moving on up!
Measures: #3

Possible key learning points

- Convert between non-adjacent metric units length from the smaller unit to the larger unit; e.g. centimetres to kilometres
- Convert between non-adjacent metric units length from the larger unit to the smaller unit; e.g. kilometres and centimetres
- Convert between non-adjacent metric units mass from the smaller unit to the larger unit; e.g. grams to kilograms
- Convert between non-adjacent metric units mass from the larger unit to the smaller unit; e.g. kilograms to grams
- Convert between non-adjacent time units; e.g. hours to seconds
- Solve problems involving converting between measures

Prerequisites

- Convert between adjacent metric units of length, mass and capacity
- Know rough equivalents between inches and cm, feet and cm, kg and lb, pint and ml
- Use decimal notation to two decimal places when converting between metric unit

Mathematical language

Length, distance
Mass, weight
Volume
Capacity
Metre, centimetre, millimetre
Tonne, kilogram, gram, milligram
Litre, millilitre
Hour, minute, second
Inch, foot, yard
Pound, ounce
Pint, gallon

Notation
Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml
Abbreviations of units in the Imperial system: lb, oz

Pedagogical notes

Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement.
The prefix 'centi-' means one hundredth, and the prefix 'milli-' means one thousandth. These words are of Latin origin.
The prefix 'kilo-' means one thousand. This is Greek in origin.
Conversion of volumes will be covered in the 'calculating space' unit.
NCETM: Glossary

Common approaches
Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)

Reasoning opportunities and probing questions

- Show me a metric (imperial) unit of measure. And another. And another.
- Kenny thinks that 2.5km = 25 000 cm. Do you agree with Kenny? Explain your answer.
- Convince me that 4.25kg does not equal 425g.

NCETM: Measurement Reasoning

Suggested activities

KM: Weighing up the options
NRICH: Place Your Orders
NRICH: Thousands and Millions
NCETM: Activity E : A little bit of history - Marco Polo

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
- Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock
- Some pupils may apply incorrect beliefs about place value, such as 2.3 × 10 = 2.30.
- Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an 'extra unit' within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.

Investigating angles — *4 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Position and direction progression map

- recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles

Return to overview

Possible themes

- Develop knowledge of angles
- Apply angle facts to deduce unknown angles

Possible key learning points

- Find missing angles where they meet at a point
- Find missing angles where they meet on a straight line
- Find missing angles where they are vertically opposite
- Solve problems involving missing angles

Prerequisites

- Know that angles are measured in degrees
- Know that angles in a full turn total 360°, and angle in half a turn must total 180°
- Estimate the size of angles

Mathematical language

Angle
Degrees
Right angle
Acute angle
Obtuse angle
Reflex angle
Protractor
Vertically opposite

Notation
Right angle notation
Arc notation for all other angles
The degree symbol (°)

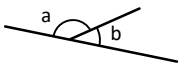
Pedagogical notes

The exact reason for there being 360 degrees in a full turn is unknown. There are various theories including it being an approximation of the 365 days in a year and resultant apparent movement of the sun, and the fact that it has so many factors.
The SI unit for measuring angles in the radian (2π radians in a full turn).
Napoleon experimented with the decimal degree, or grad (400 grads in a full turn)
NCETM: Glossary

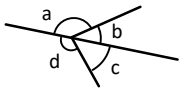
Common approaches
All pupils know how to to use a 180° and a 360° protractor.

Reasoning opportunities and probing questions

- Show a pair of possible values for a and b. And another. And another.



- Convince me that the sum of angles on a straight line is 180°.
- Show a possible set of values for a, b, c and d. And another. And another.



- Convince me that the sum of angles around a point is 360°.
- Convince me that (vertically) opposite angles are equal.
- Kenny thinks that the sum of opposite angles is 180°. Do you agree? Explain your answer.

NCETM: Geometry - Properties of Shapes Reasoning

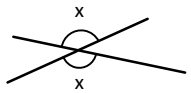
Suggested activities

KM: Maths to Infinity: Lines and Angles

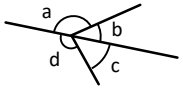
Learning review
KM: 6M10 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may think that these angles are not equal as they are not 'vertical'.



- Some pupils may think that angles that are 'roughly' opposite are always equal, e.g. a = c



Calculating fractions, decimals and percentages — 12 lessons

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Fractions, decimals and percentages progression map

- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $^1/_4 \times {}^1/_2 = {}^1/_8$]
- divide proper fractions by whole numbers [for example, $^1/_3 \div 2 = {}^1/_6$]
- multiply one-digit numbers with up to two decimal places by whole numbers
- solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison

Return to overview

Possible themes

- Calculate with fractions
- Calculate with decimals
- Calculate with percentages

Bring on the Maths⁺: Moving on up!
Fractions, decimals & percentages: #3, #4, #7
Ratio and proportion: #2

Possible key learning points

- Add fractions with different denominators
- Add a mixed number and a fraction, including with different denominators
- Add mixed numbers, including with different denominators
- Subtract fractions with different denominators
- Subtract a mixed number and a fraction, including with different denominators
- Subtract mixed numbers, including with different denominators
- Multiply a proper fraction by a proper fraction
- Divide a proper fraction by a whole number
- Multiply U.t by U
- Multiply U.th by U
- Calculate percentages of a quantity
- Solve problems involving the use of percentages to make comparisons

Prerequisites

- Convert between mixed numbers and improper fractions
- Find equivalent fractions
- Add and subtract fractions when one denominator is a multiple of the other
- Multiply a proper fraction by a whole number
- Use the formal written method of short multiplication
- Know the effect of multiplying and dividing by 10 and 100
- Know percentage equivalents of $^1/_2$, $^1/_4$, $^3/_4$, $^1/_5$, $^2/_5$, $^4/_5$

Bring on the Maths⁺: Moving on up!
Fractions, decimals & percentages: #5

Mathematical language

Mixed number
Equivalent fraction
Simplify, cancel
Lowest terms
Proper fraction, improper fraction, top-heavy fraction, vulgar fraction
Numerator, denominator
Percent, percentage

Notation
Mixed number notation
Horizontal / diagonal bar for fractions

Pedagogical notes

Use of a fraction wall to visualise multiplying fractions and dividing fractions by a whole number. For example, pupils need to read calculations such as $\frac{1}{4} \times \frac{1}{2}$ as $\frac{1}{4}$ multiplied by $\frac{1}{2}$ and therefore, $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$; $\frac{4}{10} \div 2$ as $\frac{4}{10}$ divided by 2 and therefore $\frac{2}{10}$.
NCETM: The Bar Model
NCETM: Teaching fractions
NCETM: Fractions videos
NCETM: Glossary

Common approaches
When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy.
When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy.
Pupils are encouraged to find and use 10% of an amount.

Reasoning opportunities and probing questions

- Show me an 'easy' ('difficult') pair of fractions to add (subtract). And another. And another.
- Kenny thinks that $\frac{7}{10} - \frac{2}{7} = \frac{5}{3} = 1\frac{2}{3}$. Do you agree with Kenny?
- Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain.
- Benny thinks that $\frac{4}{10} \div 2 = \frac{4}{5}$. Do you agree with Kenny? Explain.
- Lenny says *'20% of £60 is £3 because 60 ÷20 = 3'*. Do you agree?

NCETM: Fractions Reasoning
NCETM: Ratio and Proportion Reasoning

Suggested activities

NRICH: Fractions Jigsaw
NRICH: Peaches Today, Peaches Tomorrow
NRICH: Andy's Marbles
NRICH: Would you Rather?

Learning review
KM: 6M7 BAM Task, 6M8 BAM Task, 6M9 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional art of mixed numbers when adding/subtracting mixed numbers, e.g. $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$
- Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators.
- Some pupils may think that as you divide by 10 to find 10%, you divided by 15 to find 15%, divide by 20 to find 20%, etc.

Solving equations and inequalities — *4 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Algebra progression map

- enumerate possibilities of combinations of two variables
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns

Return to overview

Possible themes

- Solve missing number problems
- Understand and use algebra

Possible key learning points

- Express and solve missing number problems algebraically
- Know the basic rules of algebraic notation
- Use the basic rules of algebraic notation
- Find pairs of numbers that satisfy an equation with two unknowns e.g. a + b = 15

Prerequisites

- Use symbols to represent variables in a formula

Mathematical language

Algebra, algebraic, algebraically
Symbol
Expression
Variable
Substitute
Equation
Unknown
Enumerate

Notation
The lower case and upper case of a letter should not be used interchangeably when worked with algebra
Juxtaposition is used in place of '×'. 2*a* is used rather than *a*2.
Division is written as a fraction

Pedagogical notes

The word 'algebra' comes from the title of a book by the Persian mathematician, al-Khwārizmī, who lived in modern-day Baghdad about 1200 years ago. Al-kitāb al-mukhtaṣar fī ḥisāb **al-ǧabr** wa'l-muqābala was a book that promoted the idea of solving equations by a method of balancing.
Avoid fruit salad algebra (see possible misconceptions).
NCETM: The Bar Model
NCETM: Algebra
NCETM: Glossary

Common approaches
Use 'think of a number' problems to introduce the idea of 'n' standing for any number (a variable). For example:

- *Think of a number, double it, add 8, halve it and finally subtract your starting number.*
  The answer is always 4. $n \rightarrow 2n \rightarrow 2n + 8 \rightarrow n + 4 \rightarrow 4$.

Encourage students to try again with different numbers, including large, small, negative, fractional or decimal. Also try varying the instructions to end up with a different number every time.

Reasoning opportunities and probing questions

- $a + b = 15$. Show me a pair of values for a and b. And another. And another.
- $p + q = 7$. Show me a pair of values for p and q that no one else will think of. And another. And another.
- Kenny thinks that 'b^2' is the same as '$2b$' because when $b = 2$, $b^2 = 4$ and $2b = 4$. Do you agree with Kenny? Explain your answer.
- Jenny thinks that $7 + 2a = 9a$. Do you agree with Jenny? Explain your answer.

NCETM: Algebra Reasoning

Suggested activities

KM: Combinations of variables
NRICH: Plenty of Pens
NRICH: Your Number Is…
NRICH: Number Pyramids
NCETM: Activity A: Racetrack and Design a board game
NCETM: Activity E: Matchbox Algebra

Bring on the Maths[+]
Moving on up!: Algebra: #2

Learning review
KM: Quiz and review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may think that variables have a set value, such as $a = 1$, $b = 2$, $c = 3$, $d = 4$, etc. (especially if they have done lots of poorly designed treasure hunts/codes) – this will lead to problems such as thinking 'b^2' is the same as '$2b$' because when $b = 2$, $b^2 = 4$ and $2b = 4$.
- Using the idea of 'apples' and 'bananas' to explain $a + b = 14$ can lead to misconceptions about the use of letters as variables.
- Some students may think that the variables have to be positive integers (whole numbers)

Calculating space — 8 lessons

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Measurement and mensuration progression map

- recognise that shapes with the same areas can have different perimeters and vice versa
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3]
- recognise when it is possible to use formulae for area and volume of shape
- solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate

Return to overview

Possible themes

- Explore area
- Investigate volume
- Solve problems involving area and volume

Bring on the Maths⁺: Moving on up!
Measures: #6

Possible key learning points

- Recognise that shapes with the same areas can have different perimeters and vice versa
- Calculate the area of a parallelogram
- Calculate the area of a triangle
- Estimate the volume of cubes and cuboids
- Calculate the volume of cuboid, including cubes
- Recognise when it is possible to use formulae to calculate area and volume
- Convert between metric units of area in simple cases
- Convert between metric units of volume in simple cases

Prerequisites

- Know the meaning of perimeter (area, volume, capacity)
- Know that the area of a rectangle is given by the formula area = length × width
- Know that area can be measured using square centimetres or square metres, and the abbreviations cm^2 and m^2
- Know that volume is measured in cubes

Bring on the Maths⁺: Moving on up!
Measures: #4, #5

Mathematical language

Perimeter, area, volume, capacity
Square, rectangle, parallelogram, triangle
Composite rectilinear
Polygon
Cube, cuboid
Millimetre, Centimetre, Metre, Kilometre
Square millimetre, square centimetre, square metre, square kilometre
Cubic centimetre, centimetre cube
Formula, formulae
Convert
Length, breadth, depth, height, width

Notation
Abbreviations of units in the metric system: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

In this unit, 'volumes of shapes' refers only to cubes and cuboids.
Ensure that pupils make connections with the area of a rectangle work in Stage 5, in particular the importance of the perpendicular height.
Note that there are several different ways of stating the area of a triangle and this can cause confusion
NCETM: Glossary

Common approaches
Pupils derive practically the formulae for area of parallelogram and triangle by dissecting rectangles
Pupils derive the formula for the area of a parallelogram first. They then use this to help derive the formula for the area of an obtuse-angled triangle.
Every classroom has a set of area posters on the wall
Pupils use the area of a triangle as given by the formula area = $\frac{bh}{2}$

Reasoning opportunities and probing questions

- 'Show me' an example of when you would measure volume using km^3
- Convince me that the area of a parallelogram is found using base × height
- (Given a triangle with base labelled 8 cm, height 5 cm, slope height 6 cm) Kenny thinks that the area is 40 cm^2, Lenny thinks it is 20 cm^2, Jenny thinks it is 240 cm^2 and Benny thinks it is 24 cm^2. Who do you agree with? Explain why.

NCETM: Geometry -Properties of Shapes Reasoning

Suggested activities

KM: Fibonacci's disappearing squares
KM: Dissections deductions
KM: Stick on the Maths SSM9: Area and volume
KM: Maths to Infinity Area and Volume
NCETM: Activity C: Through the window

Learning review
KM: 6M11 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may use the sloping height when finding the areas of parallelograms and triangles
- Some pupils may think that the area of a triangle is found using area = base × height
- Some pupils may think that you multiply all the numbers to find the area of a shape

Mathematical movement — *4 lessons*

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Position and direction progression map

- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes

Return to overview

Possible themes

- Understand and use Cartesian coordinates
- Use transformations to move shapes

Possible key learning points

- Use coordinates to describe the position of a point in all four quadrants
- Use coordinates to plot the position of a point in any of the four quadrants
- Draw and translate simple shapes
- Carry out a reflection using one of the axes as a mirror line

Prerequisites

- Use coordinates in the first quadrant
- Identify a translation
- Carry out a translation in the first quadrant
- Identify a reflection
- Carry out a reflection in the first quadrant using mirror lines parallel to the axes
- Know the meaning of 'congruent', 'congruence', 'object', 'image'

Bring on the Maths⁺: Moving on up!
Properties of shapes: #2
Position and direction: #1

Mathematical language

2-D
Grid
Axis, axes, x-axis, y-axis
Origin
Quadrant
(Cartesian) coordinates
Point
Translation
Reflection
Transformation
Object, Image
Congruent, congruence

Notation
Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y)

Pedagogical notes

The main focus of this unit is to develop understanding of coordinates in all four quadrants
Note that pupils are not yet expected to use an algebraic description of a mirror line (such as x = 3).
The French mathematician Rene Descartes introduced Cartesian coordinates in the 17th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling.
Other coordinate systems include grid references, polar coordinates and spherical coordinates.
There are other types of mathematical movement that pupils will learn about in future stages. The group name for these movements is 'transformations'.
NCETM: Glossary

Common approaches
Teachers do not use the phrase 'along the corridor and up the stairs' as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is 'x is a cross, wise up!'
Teachers use the language 'negative number', and not 'minus number', to avoid future confusion with calculations.

Reasoning opportunities and probing questions

- (Given a grid with the point (-3, 4) indicated) Benny describes this point as (-3, 4). Jenny describes the point as (4, -3). Who do you agree with? Why?
- Two vertices of a rectangle are (-1, 2) and (4, -2). What could the other two vertices be? How many solutions can you find?
- Convince me that (-2, 3) is in the second quadrant)

NCETM: Geometry: Position Direction and Movement Reasoning

Suggested activities

KM: Stick on the Maths ALG2: Coordinates in four quadrants
NRICH: Cops and Robbers
NRICH: Eight Hidden Squares
NRICH: Coordinate Tan
NRICH: Transformation Tease
NCETM: Activity B - Battleships

Learning review
KM: 6M12 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
- When reflecting a triangle some students may draw a translation
- When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance form the mirror line.
- Some pupils will confuse the order of x-coordinates and y-coordinates
- When constructing axes, some pupils may not realise the importance of equal divisions on the axes

Presentation of data	4 lessons

Key concepts (Upper Key Stage 2 National Curriculum statements)

The Big Picture: Statistics progression map

- interpret and construct pie charts and line graphs and use these to solve problems

Return to overview

Possible themes

- Construct and interpret pie charts
- Solve problems involving graphs and charts

Bring on the Maths+: Moving on up!
Statistics: #2, #3

Possible key learning points

- Interpret pie charts
- Construct a pie chart by measuring angles
- Interpret line graphs
- Construct line graphs

Prerequisites

- Measure and construct angles using a protractor
- Interpret and construct a simple line graph

Mathematical language

Data
Scale
Axis, axes
Graph
Frequency
Time graph, Time series
Line graph
Pie chart
Sector
Angle
Protractor
Degrees
Maximum, minimum

Pedagogical notes

In Stage 6, when constructing pie charts the total of the frequencies is always a factor of 360 More complex cases are included in later stages.

William Playfair, a Scottish engineer and economist, introduced the line graph in 1786. He also introduced the pie chart in 1801.

NCETM: Glossary

Common approaches
Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency, and not using percentages.
The angle for the first section is measured from a vertical radius. Subsequent sections are measured using the boundary line of the previous section.

Reasoning opportunities and probing questions

- Show me a pie chart representing the following information: Blue (25%), Red (over 50%), Yellow (the rest). And another. And another.
- Always / Sometimes / Never: Pie charts are constructed in a clockwise direction
- Always / Sometimes / Never: The larger the size of the pie chart, the greater the total frequency
- Kenny says 'If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.' Do you agree with Kenny? Explain your answer.

NCETM: Statistics Reasoning

Suggested activities

KM: Stick on the Maths HD6: Graphs and diagrams
NRICH: Match the Matches
NRICH: Graphing Number Patterns
NCETM: A little bit of history (Britain since 1945)

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may think that a line graph is appropriate for discrete data
- Some pupils may think that each square on the grid used represents one unit
- Some pupils may confuse the fact that the sections of the pie chart total 100% and 360°

Measuring data	4 lessons

Key concepts (Upper Key Stage 2 National Curriculum statements)

- calculate and interpret the mean as an average

The Big Picture: Statistics progression map

Return to overview

Possible themes	Possible key learning points
• Understand and use the mean **Bring on the Maths+: Moving on up!** Statistics: #4	• Understand the meaning of 'average' as a typicality (or location) • Calculate the mean of a set of discrete data • Interpret the mean of a set of discrete data • Use the mean to find a missing number in a set of data

Prerequisites	Mathematical language	Pedagogical notes
• Approximate a number by rounding to a given number of decimal places	Average Mean Measure Data Statistic Statistics Approximate Round	The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean'). Other types of average, including the mode and the median, are introduced in later stages. NCETM: Glossary **Common approaches** *Always use brackets when writing out the calculation for a mean, e.g. (2 + 3 + 4 + 5) ÷ 4 = 14 ÷ 4 = 3.5*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Always / Sometimes / Never: The mean is a whole number. • Kenny is working out the mean of 2, 3, 4 and 5. He calculates 2 + 3 + 4 + 5 ÷ 4 = 10.25. Do you agree with Kenny? Explain your answer. • The average number of children per family (Married Couples, 2012) is 1.8. Convince me that this statement makes sense. NCETM: Statistics Reasoning	KM: Maths to Infinity: Averages, Charts and Tables NRICH: Birdwatch NRICH: Probably … NRICH: Same or Different? NCETM: A little bit of history (Britain since 1945) **Learning review** KM: 6M13 BAM Task NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• If using a calculator some pupils may not use the '=' symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as 2 + 3 + 4 + 5 ÷ 4 = 10.25. • Some pupils may think the average is always the middle number • Some pupils may think that the mean must be a whole number • Some pupils may not realise that the mean must lie within the range of the data set.